

# 1 Section 5.5 Graphs of the Sine and Cosine Functions

## The graph of the sine function

**Example 1** Graph  $y = \sin x$ ,  $0 \leq x \leq 2\pi$  by plotting points.

Because the domain of the sine function is the real numbers and the period is  $2\pi$ , the graph of  $y = \sin x$  is drawn by repeating the portion in the last figure.

Note that the maximum value ( $M$ ) is 1 and the minimum value ( $m$ ) is  $-1$ .

The **amplitude** of the graph of  $y = \sin x$  is given by  $\text{Amplitude} = \frac{1}{2}(M - m)$ .

The amplitude of  $y = a \sin x$  is  $|a|$ .

**Example 2** Determine the amplitude of each of the following function  $y = -5 \sin x$ .

**Example 3** Graph  $y = 3 \sin x$ .

One cycle of  $y = \sin bx$  is completed as  $bx$  varies from 0 to  $2\pi$ . The **period** of  $y = \sin bx$  is  $\frac{2\pi}{|b|}$ .

**Example 4** Determine the period of the following function  $y = \sin 4x$ .

One cycle of the graph of  $y = a \sin bx$  for both  $a$  and  $b$  positive has the following.

1. The amplitude is  $a$ .
2. The period is  $\frac{2\pi}{b}$ .
3. For  $0 \leq x \leq \frac{2\pi}{b}$ , the zeros are  $0, \frac{\pi}{b}, \frac{2\pi}{b}$ .
4. The maximum value is  $a$  when  $x = \frac{\pi}{2b}$ , and the minimum value is  $-a$  when  $x = \frac{3\pi}{2b}$ .
5. If  $a < 0$ , the graph is reflected across the x-axis.

**Example 5** Graph  $y = -2 \sin \frac{1}{2}x$  on  $[0, 8]$ .

## The graph of the cosine function

**Example 6** Graph  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ .

The period is  $2\pi$  and so the graph of  $y = \cos x$  can be drawn by repeating the portion in the last figure.

The amplitude of  $y = a \cos x$  is  $|a|$ .

The period of  $y = \cos bx$  is  $\frac{2\pi}{|b|}$ .

One cycle of the graph of  $y = a \cos bx$  for both  $a$  and  $b$  positive has the following.

1. The amplitude is  $a$ .
2. The period is  $\frac{2\pi}{b}$ .
3. For  $0 \leq x \leq \frac{2\pi}{b}$ , the zeros are  $\frac{\pi}{2b}, \frac{3\pi}{2b}$ .
4. The maximum value is  $a$  when  $x = 0$ , and the minimum value is  $-a$  when  $x = \frac{\pi}{b}$ .
5. If  $a < 0$ , the graph is reflected across the x-axis.

**Example 7** Graph  $y = -3 \cos 2\pi x$  on  $[0, 6]$ .

**Example 8** Graph  $y = |\cos x|$  when  $0 \leq x \leq 2\pi$ .

**Example 9** Graph  $y = \sin |x|$ .

**Example 10** Find an equation of the following graphs

#### Translation of Trigonometric Functions

The graph of  $y = f(x) - c$  is shifted  $c$  units down from the graph of  $y = f(x)$ .  
 The graph of  $y = f(x) + c$  is shifted  $c$  units up from the graph of  $y = f(x)$ .

The graph of  $y = f(x - d)$  is shifted  $d$  units to the right from the graph of  $y = f(x)$ . The graph of  $y = f(x + d)$  is shifted  $d$  units to the left from the graph of  $y = f(x)$ .

The graph of  $y = a \sin(bx + c)$  and  $y = a \cos(bx + c)$ , with  $b > 0$  have

Amplitude:  $|a|$       Period:  $\frac{2\pi}{b}$       **Phase Shift:**  $-\frac{c}{b}$

One cycle of each graph is completed on the interval  $-\frac{c}{b} \leq x \leq -\frac{c}{b} + \frac{2\pi}{b}$ .

**Example 11** Graph the following: 1)  $y = 2 \cos(3x - 2)$       2)  $y = 2 \sin(4x + \pi) + 1$       3)  $y = -2 \cos(\pi x + \frac{\pi}{2}) + 1$

**Example 12** Find an equation of the following graphs