

# 1 Section 5.2 Trigonometric Functions of acute Angles

When we are working with right triangles, it is convenient to refer to the side opposite an angle or the side adjacent to (next to) an angle.

Let  $\theta$  be an acute angle of a right triangle. The values of the **six trigonometric functions** of  $\theta$  are

$$\begin{aligned} \sin \theta &= \frac{\text{length of opposite side}}{\text{length of hypotenuse}} & \cos \theta &= \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} & \tan \theta &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ \cot \theta &= \frac{\text{length of adjacent side}}{\text{length of opposite side}} & \sec \theta &= \frac{\text{length of hypotenuse}}{\text{length of adjacent side}} & \csc \theta &= \frac{\text{length of hypotenuse}}{\text{length of opposite side}} \end{aligned}$$

**Example 1** Find the value of the six trigonometric functions of  $\beta$  for the following triangle.

**Example 2** Given  $\tan \theta = \frac{2}{3}$ , find  $\cos \theta$ .

## Trigonometric Functions of Special Angles

For some special angles, the value of a trigonometric functions of the angles can be found by geometric methods. These special acute angles are  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ . Let us begin with  $45^\circ$ . Because  $\angle A = \angle B$ , the lengths of the sides opposite these angles are equal. Let the length of each equal side be denoted by  $a$ . Now  $r = a\sqrt{2}$ . The values of the six trigonometric functions of  $45^\circ$  are  $\sin 45^\circ = \frac{\sqrt{2}}{2}$ ,  $\cos 45^\circ = \frac{\sqrt{2}}{2}$ ,  $\tan 45^\circ = 1$ ,  $\cot 45^\circ = 1$ ,  $\sec 45^\circ = \sqrt{2}$ , and  $\csc 45^\circ = \sqrt{2}$ .

Now, for  $30^\circ$  and  $60^\circ$ . If we draw an equilateral triangle and bisect one of the angles. The angle bisector also bisects one of the sides. The length of the side opposite to the  $30^\circ$  angle is one-half the length of the hypotenuse of triangle  $OAB$ . Let  $a$  denote the length of the hypotenuse. Then the length of the side opposite the  $30^\circ$  angle is  $\frac{a}{2}$ . Then  $h = \frac{\sqrt{3}}{2}a$ . The values of the six trigonometric functions of  $30^\circ$  are  $\sin 30^\circ = \frac{1}{2}$ ,  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ ,  $\tan 30^\circ = \frac{\sqrt{3}}{3}$ ,  $\cot 30^\circ = \sqrt{3}$ ,  $\sec 30^\circ = \frac{2\sqrt{3}}{3}$ , and  $\csc 30^\circ = 2$ .

The values of the six trigonometric functions of  $60^\circ$  are  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ ,  $\cos 60^\circ = \frac{1}{2}$ ,  $\tan 60^\circ = \sqrt{3}$ ,  $\cot 60^\circ = \frac{\sqrt{3}}{3}$ ,  $\sec 60^\circ = 2$ , and  $\csc 60^\circ = \frac{2\sqrt{3}}{3}$ .

## Trigonometric Functions of Special Angles

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
$30^\circ; \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ; \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ; \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

**Example 3** Find the exact value of  $\sin^2 60^\circ + \cos^2 30^\circ$ .

**Example 4** Find  $\sin \frac{\pi}{3} \cos \frac{\pi}{4} - \tan \frac{\pi}{4}$ .

It is clear that the sine and cosecant functions are called **reciprocal functions**. Also, the cosine and secant are also reciprocal functions, as are the tangent and cotangent functions.

If  $\sin \alpha = \frac{2}{3}$ , find  $\csc \alpha$ .

#### **Applications Involving Right Triangles**

In some applications, a horizontal line of sight is used as a reference. an angle measured above the line of sight is called an **angle of elevation** and angle measured below the angle of sight is called an **angle of depression**.

**Example 5** *From a point 10 feet from the base of a flag pole, the angle of elevation to the top of the flag pole is  $45^\circ$ . Find the height of the flag pole.*

**Example 6** *If the distance from a helicopter to a tower is 300 feet and the angle of depression is  $60^\circ$ . Find the distance on the ground from a point directly below the helicopter to the tower.*

**Example 7** *A backpacker notes that from a certain point on level ground, the angle of elevation to a point at the top of a tree is  $30^\circ$ . After walking 50 feet closer to the tree, the backpacker notes that the angle of elevation is  $60^\circ$ . Find the height of the tree.*