1 Section 5.1 Angles and Arcs

A point P on a line separates the line into two parts, each of which is called a half-line. The union of point P and the half-line formed by P that includes point A is called a ray, and it is represented as $\overrightarrow{PA}$. The point P is the endpoint of ray $\overrightarrow{PA}$.

An **angle** is formed by rotating a given ray about its endpoint to some terminal position. The original ray is the **initial side** of the angle, and the second ray is the **terminal side** of the angle. The common endpoint is the **vertex** of the angle.

It can be named as $\angle A$, $\angle O$, $\angle AOB$, or $\angle BOA$. Angles formed by a counterclockwise rotation are considered **positive angles**, and angles formed by a clockwise rotation are considered **negative angles**.

The measure of an angle is determined by the amount of rotation of the initial ray. One **degree** is the measure of an angle formed by rotating a ray $\frac{1}{360}$ of a complete revolution. the symbol for degree is $^\circ$.

Angles are often classified according to their measure.

180° angles are **straight angles**.

90° angles are **right angles**.

Angles that have a measure greater than 0° but less than 90° are **acute angles**.

Angles that have a measure greater than 90° but less than 180° are **obtuse angles**.

An angle is in **standard position** if its vertex is at the origin and its initial side is on the positive x-axis.

Two positive angles are **complementary angles** if the sum of the measures of the angles is 90°.

Two positive angles are **supplementary angles** if the sum of the measures of the angles is 180°.

Example 1 *Classify each of the following angles as acute, right, obtuse, or straight:* 1)90° 2)180° 3)15° 4)150°.

Example 2 *For each angle, find the measure (if possible) of its complement and of its supplement:* 1)50° 2)120°.

Some angles have a measure greater than 360° and some angles have a measure less than -360°.

If the terminal side of an angle in standard position lies on a coordinate axis, then the angle is classified as a **quadrantal angle**. For example, 90°, 180°, 270° are all quadrantal angles.

If the terminal side of an angle in standard position does not lie on a coordinate axis, then the angle is classified according to the quadrant that contains the terminal side. for example 150° is a Quadrant II angle.

Example 3 *For each of the following angles either state that it is a quadrantal angle or state which quadrant contains its terminal side. An angle in standard position that measures:* 1)45° 2)90° 3)120° 4)180° 5)345°.

Example 4 *True or False: The supplement of an obtuse angle is always an acute angle.*

Angles in standard position that has the same terminal sides are coterminal angles.

Example 5 *True or False: Every angle has an unlimited number of coterminal angles.*

Measures of Coterminal Angles

Given $\angle\theta$ in standard position with measure x°, then the measures of the angles that are **coterminal** with $\angle\theta$ are given by $x^\circ + k \cdot 360^\circ$ where k is an integer.

Example 6 *If* $\theta = 430^\circ$, *then* $\angle 1 = 430^\circ + (-1) \cdot 360 = 70^\circ$, $\angle 2 = 430^\circ + (-2) \cdot 360 = -290^\circ$, *and* $\angle 3 = 430^\circ + (1) \cdot 360 = 790^\circ$ *are all coterminal angles.*

Example 7 *Assume that the following angles are in standard position. Classify each angle by quadrant, and then determine the measure of the positive angle with measure less than* 360° *that is coterminal with the given angle.* 1)α *measures* 400° 2)β *measures* -130° 3)γ *measures* 1000°.

There are two popular methods for representing a fractional part of a degree. 1) Decimal Degree Method 29.76° means 29° plus 76 hundredths of 1°. 2)DMS (Degree, Minute, Second).

In the DMS method, a degree is subdivided into 60 equal parts, each of which is called a **minute**, denoted by $'$. Thus $1^\circ = 60'$. Also, a minute is subdivided into 60 equal parts, each of which is called a **second**, denoted by ". Thus $1' = 60$" and $1^\circ = 3600$".

The **conversion factors** are $\frac{1^\circ}{60'} = 1$, $\frac{1'}{60"} = 1$ and $\frac{1^\circ}{3600"} = 1$.

Example 8 *Write the degree* $126^\circ\ 12'\ 27"$ *in terms of the decimal degree method.*

Example 9 *Find the complement of an angle that measures* $36^\circ\ 43'\ 29"$.

Radian Measure

We can measure angles by radians. One **radian** is the measure of the central angle subtended by an arc of length r on a circle of radius r.

Given an arc of length s on a circle of radius r, the measure of the central angle subtended by the arc is $\theta = \frac{s}{r}$ radians.

Radian is a dimensionless quantity because there are no units of measurement associated with a radian.

Since the circumference of a circle is $C = 2\pi r$, then the radian measure of the central angle subtended by the circumference is $\theta = \frac{2\pi r}{r} = 2\pi$. Since the central angle θ subtended by the circumference, in degree measures, is 360°, then $360^\circ = 2\pi$ radians and so $180^\circ = \pi$ radians.

To convert from radians to degrees, multiply by $\left(\frac{180^\circ}{\pi \text{ radinas}}\right)$

To convert from degrees to radians, multiply by $\left(\frac{\pi \text{ radians}}{180^\circ}\right)$.

Example 10 *Which is larger,* 1 *radian or* 1°?

Example 11 *Which is larger,* π *radians or* π°?

Example 12 *Convert* 450° *to radians and convert* $-\frac{3}{4}\pi$ *radians to degrees.*

Example 13 *Find the supplement of an angle that measures* $\frac{\pi}{4}$ *radians.*

Arc and Arc Length

Let r be the length of the radius of a circle and θ the nonnegative radian measure of a central angle of the circle. Then the **length of the arc** s that subtends the central angle is $s = r\theta$.

Example 14 *Find the length of an arc that subtends a central angle of* 120° *in a circle of radius* 10 *cm.*

Linear and Angular Speed

Linear speed v is distance traveled per unit time, i.e., $v = \frac{s}{t}$ where v is the linear speed, s is the distance travelled,and t is the time.

Angular speed ω is the angle through which a point on a circle moves per unit time, i.e., $w = \frac{\theta}{t}$ where w is the angular speed, θ is the measure (in radians) of the angle through which a point has moved, and t is the time. Some common units of w are revolutions per second, revolutions per minute, radians per second, and radians per second.

Example 15 *The minute hand on a clock rotates at* 1 *revolutions per hour. Find the angular speed of the minute hand of the clock in radians per second.*

There is a relation between the linear speed v and the angular speed w and that relation is $v = rw$, where r is the radius of the circle.

Example 16 *A wheel on a bicycle has a radius of* 12 *inches. The bicycle wheel is rotating at* 140 *revolutions per minute. Find the speed of the bicycle to the nearest mile per hour.*