1 Section 4.4 Properties of Logarithms

Basic Properties of Logarithms

1) $\log_b b = 1$ 2) $\log_b 1 = 0$ 3) $\log_b (b^p) = p$

Properties of Logarithms

In the following properties, b, M, N are positive real numbers $(b \neq 1)$, and p is any real number.

Product property $\log_b MN = \log_b M + \log_b N$
Quotient property $\log_b \frac{M}{N} = \log_b M - \log_b N$.
Power property $\log_b (M^p) = p \log_b M$.
One-to-one property $\log_b M = \log_b N$ implies $M = N$.
Logarithm-of-each-side property $M = N$ implies $\log_b M = \log_b N$.
Inverse property $b^{\log_b P} = p$ (for $p > 0$) .

Example 1 *If* $\log_b 2 = 0.5$ *and* $\log_b 3 = 0.8$, *evaluate* $\log_b 72$.

Example 2 *True or False*

1. $\log_b 2 + \log_b 3 = \log_b 5$.
2. $\log_b 4 - \log_b 3 = 1$.
3. $\log_b 4 + \log_b 3 = \log_b 12$.
4. $2 \log_b 2 + \log_b 3 = \log_b 12$.
5. $\frac{1}{2} \log_b x = \log_b \sqrt{x}$, $x > 0$.
6. $\log_b x - \log_b y + \log_b z = \log_b \frac{x}{yz}$.

Example 3 *Use the properties of logarithms to express the following logarithms in terms of logarithms of x, y, and z.* 1) $\log_b \left(5x^3yz^4\right)$ 2) $\log_b \frac{2y^3}{x^2\sqrt{z}}$

Example 4 *Use the properties of logarithms to rewrite each expression as a single logarithm with a coefficient of* 1. 1) $\frac{1}{2} \log_b z - 3 \log_b y - 2 \log_b (x+5)$ 2) $3 \log_b (y-2) - 2 \log_b (y-3)$

Example 5 *True or False:* 1) $\log_b (2+3) = \log_b 2 + \log_b 3$ 2) $\log_b (1+2+3) = \log_b 1 + \log_b 2 + \log_b 3$ 3) $\frac{\log_b 12}{\log_b 3} = \log_b 4$.

Example 6 *Given* $\log_6 2 \simeq 0.3869$, $\log_6 5 \simeq 0.8982$, *and* $\log_6 7 \simeq 1.0860$, *evaluate* 1) $\log_6 140$ 2) $\log_6 \frac{7}{8}$ 3) $\log_6 \sqrt[4]{98}$

Change-of-base Formula

If x, a, and b are positive real numbers with $a \neq 1$, and $b \neq 1$, then $\log_b x = \frac{\log_a x}{\log_a b}$.

Note that $\log_b x = \frac{\log x}{\log b}$ or $\log_b x = \frac{\ln x}{\ln b}$.

Example 7 *Evaluate each algorithm:* 1) $\log_4 150$ 2) $\log_{16} 1000$

Example 8 *True or False. (Assume* $x > 0$*)* 1) $\log_5 x = \frac{\log x}{5}$ 2) $\log_5 x = \frac{\ln x}{\ln 5}$

Example 9 *Evaluate* $\log_3 5 \cdot \log_5 7 \cdot \log_7 9$.

Logarithmic Inequalities

In the following b, x are positive real numbers ($b \neq 1$).

If $b > 1$, then $\log_b x \leq a$ implies that $x \leq b^a$. However, if $0 < b < 1$, then $\log_b x \leq a$ implies that $x \geq b^a$.

Example 10 *Find the solution set (in interval notation) of the following inequalities:* 1) $\ln x \leq 3$ 2) $-1 \leq \log_{\frac{1}{2}} x \leq 3$.