1 Section 4.3 Logarithmic Functions and Their Graphs

$g(x) = b^x$ is a one-to-one function and therefore has an inverse function.

Definition of Logarithm

If $x > 0$ and b is a positive constant $(b \neq 1)$, then $y = \log_b x$ if and only if $b^y = x$.

The notation $\log_b x$ is read " the logarithm (or log) base b of x." The function $f(x) = \log_b x$ is a **Logarithmic function with base** b. This function is the inverse of the Exponential function $g(x) = b^x$.

Inverse of Logarithmic and Exponential Functions

Let $g(x) = b^x$ and $f(x) = \log_b x$ $(x > 0,\ b > 0,\ b \neq 1)$. Then $g(f(x)) = b^{\log_b x} = x$ and $f(g(x)) = \log_b b^x = x$.

The **exponential form** of $y = \log_b x$ is $b^y = x$.

The **logarithmic form** of $b^y = x$ is $y = \log_b x$.

Example 1 *Write each equation in its exponential form* $1) 5 = \log_4 x$ $\quad 2) 4 = \log_6(x-1)$ $\quad 3) \log_3 81 = x$.

Example 2 *Write each equation in its logarithmic form* $1) 3^2 = 9$ $\quad 2) 5^3 = x$ $\quad 3) a^b = c$.

Basic Logarithmic Properties

$1) \log_b b = 1$ $\quad 2) \log_b 1 = 0$ $\quad 3) \log_b(b^p) = p$

Example 3 *Evaluate each of the following:*

$1) \log_4 4$ $\quad 2) \log_5 125$ $\quad 3) \log_5 1$ $\quad 4) \log_7 \frac{1}{49}$ $\quad 5) \log_{0.5} 16$

Graphs of Logarithmic Functions

The graph of $f(x) = \log_b x$ is a reflection of the graph $g(x) = b^x$ with respect to the line $y = x$. $\left(f(x) = g^{-1}(x)\right)$

Example 4 *Graph* $1) f(x) = \log_5 x$ $\quad 2) g(x) = \log_{\frac{2}{3}} x$

Properties of $f(x) = \log_b x$

For all positive real numbers b, $b \neq 1$, the function defined by $f(x) = \log_b x$ has the following properties:

1. f has the set of positive real numbers as its domain.
2. f has the set of real numbers as its range.
3. f has a graph with an x-intercept of $(1, 0)$.
4. f has a graph with no y-intercept.
5. f is a one-to-one function.

6. f has a graph asymptotic to the y-axis. If $b > 1$, $f(x) \longrightarrow -\infty$ as $x \longrightarrow 0^+$. If $0 < b < 1$, $f(x) \longrightarrow \infty$ as $x \longrightarrow 0^+$.

7. f is an increasing function if $b > 1$ and f is a decreasing function if $0 < b < 1$.

Domains of Logarithmic Functions
If $f(x) = \log_b g(x)$, then $b \neq 1$, $b > 0$, and $g(x) > 0$.

Example 5 *Find the domain of the following logarithmic functions:* $1) f(x) = \log_4(x+4)$ $2) f(x) = \log_8|x-3|$ $3) f(x) = \log_2\left(\frac{2x}{x+4}\right)$ $4) f(x) = \log_x(1-x)$

Example 6 *Use translations to graph* $1) f(x) = \log_4(x+3)$ $2) f(x) = \log_{\frac{1}{2}} x + 3$ $3) f(x) = 1 + \log_{\frac{5}{2}}(x-4)$

Common and Natural Logarithms

The function defined by $f(x) = \log_{10} x$ is called the **common logarithmic function**. It is customarily written without the base as $f(x) = \log x$.

The function defined by $f(x) = \log_e x$ is called the **natural logarithmic function**. It is customarily written as $f(x) = \ln x$.