

1 Section 4.3 Logarithmic Functions and Their Graphs

$g(x) = b^x$ is a one-to-one function and therefore has an inverse function.

Definition of Logarithm

If $x > 0$ and b is a positive constant ($b \neq 1$), then $y = \log_b x$ if and only if $b^y = x$.

The notation $\log_b x$ is read "the logarithm (or log) base b of x ." The function $f(x) = \log_b x$ is a **Logarithmic function with base b** . This function is the inverse of the Exponential function $g(x) = b^x$.

Inverse of Logarithmic and Exponential Functions

Let $g(x) = b^x$ and $f(x) = \log_b x$ ($x > 0$, $b > 0$, $b \neq 1$). Then $g(f(x)) = b^{\log_b x} = x$ and $f(g(x)) = \log_b b^x = x$.

The **exponential form** of $y = \log_b x$ is $b^y = x$.

The **logarithmic form** of $b^y = x$ is $y = \log_b x$.

Example 1 Write each equation in its exponential form 1) $5 = \log_4 x$ 2) $4 = \log_6(x - 1)$ 3) $\log_3 81 = x$.

Example 2 Write each equation in its logarithmic form 1) $3^2 = 9$ 2) $5^3 = x$ 3) $a^b = c$.

Basic Logarithmic Properties

1) $\log_b b = 1$ 2) $\log_b 1 = 0$ 3) $\log_b(b^p) = p$

Example 3 Evaluate each of the following:

1) $\log_4 4$ 2) $\log_5 125$ 3) $\log_5 1$ 4) $\log_7 \frac{1}{49}$ 5) $\log_{0.5} 16$

Graphs of Logarithmic Functions

The graph of $f(x) = \log_b x$ is a reflection of the graph $g(x) = b^x$ with respect to the line $y = x$. ($f(x) = g^{-1}(x)$)

Example 4 Graph 1) $f(x) = \log_5 x$ 2) $g(x) = \log_{\frac{2}{3}} x$

Properties of $f(x) = \log_b x$

For all positive real numbers b , $b \neq 1$, the function defined by $f(x) = \log_b x$ has the following properties:

1. f has the set of positive real numbers as its domain.
2. f has the set of real numbers as its range.
3. f has a graph with an x-intercept of $(1, 0)$.
4. f has a graph with no y-intercept.
5. f is a one-to-one function.

6. f has a graph asymptotic to the y-axis. If $b > 1$, $f(x) \rightarrow -\infty$ as $x \rightarrow 0^+$. If $0 < b < 1$, $f(x) \rightarrow \infty$ as $x \rightarrow 0^+$.
7. f is an increasing function if $b > 1$ and f is a decreasing function if $0 < b < 1$.

Domains of Logarithmic Functions

If $f(x) = \log_b g(x)$, then $b \neq 1$, $b > 0$, and $g(x) > 0$.

Example 5 Find the domain of the following logarithmic functions: 1) $f(x) = \log_4(x+4)$ 2) $f(x) = \log_8|x-3|$ 3) $f(x) = \log_2\left(\frac{2x}{x+4}\right)$ 4) $f(x) = \log_x(1-x)$

Example 6 Use translations to graph 1) $f(x) = \log_4(x+3)$ 2) $f(x) = \log_{\frac{1}{2}}x+3$ 3) $f(x) = 1 + \log_{\frac{3}{2}}(x-4)$

Common and Natural Logarithms

The function defined by $f(x) = \log_{10} x$ is called the **common logarithmic function**. It is customarily written without the base as $f(x) = \log x$.

The function defined by $f(x) = \log_e x$ is called the **natural logarithmic function**. It is customarily written as $f(x) = \ln x$.