

1 Section 4.2 Exponential Functions and Their Graphs

Definition of an Exponential Function

The exponential function f with base b is defined by $f(x) = b^x$ where $b > 0$, $b \neq 1$, and x is any real number.

Example 1 Identify whether each of the following functions is an exponential function.

1) $f(x) = 3x$ 2) $f(x) = 3^x$ 3) $f(x) = x^3$

If the base of an exponential function were a negative number, the value of the function should be a complex number for some values of x . (Check $f(\frac{1}{2})$ where $f(x) = (-4)^x$).

If $b = 1$, then $1^x = 1$ for all values of x .

If $b = 0$, then $0^x = 0$ for $x > 0$ and is undefined for $x \leq 0$.

Example 2 Examples of Exponential Functions are $f(x) = 2^x$, $g(x) = (\frac{2}{3})^x$, and $h(x) = \pi^x$.

Example 3 Find the value of 1) $f(3)$ 2) $g(-2)$

Example 4 Find $f(\sqrt{5})$ when $f(x) = 4^x$. (By calculator $4^{\sqrt{5}} \approx 4^{2.236068} \approx 22.194587$)

Graphs of Exponential Functions

Example 5 Graph $f(x) = 2^x$ by plotting points.

Example 6 Graph $f(x) = (\frac{1}{2})^x$ by plotting points.

Properties of $f(x) = b^x$

For positive real numbers b , $b \neq 1$, the exponential function defined by $f(x) = b^x$ has the following properties:

1. f has the set of real numbers as its domain.
2. f has the set of positive real numbers as its range.
3. f has a graph with y-intercept of $(0, 1)$.
4. f has no x-intercept.
5. f is a one-to-one function.
6. f has a graph asymptotic to the x-axis. If $b > 1$, $f(x) \rightarrow 0$ as $x \rightarrow -\infty$.
If $0 < b < 1$, $f(x) \rightarrow 0$ as $x \rightarrow \infty$.
7. f is an increasing function if $b > 1$. f is a decreasing function if $0 < b < 1$.

Example 7 Sketch the graph of each function 1) $f(x) = \left(\frac{1}{3}\right)^x + 2$ 2) $g(x) = \left(\frac{4}{3}\right)^{x+2}$ 3) $f(x) = 2^{|x|}$

The Natural Exponential Function

It can be shown that as $n \rightarrow \infty$, $\left(1 + \frac{1}{n}\right)^n \rightarrow e$. The value of e accurate to eight places is 2.71828183.

For all real numbers x , the function defined by $f(x) = e^x$ is called the **natural exponential function**.

Example 8 Graph $f(x) = e^x$.

Example 9 Graph $g(x) = e^{-x}$ and $h(x) = -e^x$ on the same axis.