1 Section 4.1 Inverse Functions

Definition of an Inverse Function

If f is a one-to-one function with domain X and range Y, and g is a function with domain Y and range X, then g is the inverse function of f if and only if $(f \circ g)(x) = x$ for all x in the domain of g and $(g \circ f)(x) = x$ for all x in the domain of f.

Example 1 *Verify that* $h(x) = \frac{1}{4}x + 8$ *is the inverse function of* $g(x) = 4x - 32$.

Example 2 *A function* f *is given as the set* $\{(1,2),(3,4),(5,6)\}$

1) Is f a one-to-one function?
2) What is the inverse function of f?

Example 3 *A function* g *is given as the set* $\{(1,2),(3,4),(5,4)\}$

1) Is g a one-to-one function?
2) Is there an inverse function of g? why?
3) Is there an inverse relation of g?

A function f has an inverse function if and only if it is a one-to-one function.

Example 4 *If the function* $f(x)$ *is a one-to-one function and* $f(2) = 11$*, then find* $f^{-1}(11)$.

Example 5 *A one-to-one function* f *has domain* $[-3,5]$ *and range* $[-1,2]$

1)What is the domain of the inverse function of f?
1)What is the range of the inverse function of f?

Find the Equation for f^{-1}

To find the inverse f^{-1} of the one-to-one function f:

Substitute y for $f(x)$.

Interchange x and y.

Solve, if possible, for y in terms of x.

Substitute $f^{-1}(x)$ for y.

Verify that the domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.

Example 6 *Find the inverse of the following one-to-one functions:*

1)$f(x) = 3x + 1$ 2)$g(x) = \frac{2x+1}{2x-1}$

Place a Restriction on a Domain

Example 7 *The function* $f(x) = |x|$ *is not a one-to-one function. What restrictions could be placed on the domain of* f *that will allow* f *to be a one-to-one function.*

Example 8 *Find the inverse* f^{-1} *of the function* $f(x) = x^2 + 6x$,*for* $x \geq 3$ *and state the domain and the range of both* f *and* f^{-1}.

Example 9 *Find the inverse* f^{-1} *of the function* $f(x) = \sqrt{16 - x^2}$,*for* $-4 \leq x \leq 0$ *and state the domain and the range of both* f *and* f^{-1}.

Graphs of Inverse Functions

The graph of a function f and the graph of the inverse function f^{-1} are symmetric with respect to the line given by $y = x$.

Example 10 *Sketch the graph of* f^{-1}, *given that* f *is the function shown in he figure.*

Example 11 *Graph* $g(x) = x^2 - 2x$ *for* $x \geq 1$, *and its inverse function.*

Example 12 *If* $f(x) = x^2 + 1$ *with domain* $(-\infty, 0]$, *then find* $f^{-1}(x)$.

Example 13 *Let* $f(x) = -x^2 - 3x + k$ *and* $f^{-1}(x)$ *exists. If* $f^{-1}(2) = 3$, *then find the value of* k.

Example 14 *If* $f(x) = \frac{2x+1}{x-3}$, *then find the value of* $\left(f \circ f^{-1}\right)(5) + f^{-1}(1)$.

Example 15 *If* $f(x) = 2x - 1$, *then find the value of* $\left(f^{-1} \circ f^{-1} \circ f\right)(3)$.

Example 16 *Which one of the following statements is FALSE?*

1) In order for a function to have an inverse, it must be one-to-one. 2) The domain of f equals the range of f^{-1}. 3) If the point (a, b) lies on the graph of f and f has an inverse, then the point (b, a) lies on the graph of f^{-1}. 4)If a function f has an inverse and $f(-3) = 6$, then $f(6) = -\frac{1}{3}$. 5) If a function f has an inverse, then the graph of f^{-1} may be obtained by reflecting the graph of f across the line with equation $y = x$.