1 Section 3.2 Polynomial Functions

In Chapter 2, we talked about the following ($a \neq 0$):

$1) P(x) = a$, (Degree 0). Its graph is a horizontal line through $(0, a)$.

$2) P(x) = ax + b$ (Degree 1). Its graph is a line with y-intercept $(0, b)$ and slope a.

$3) P(x) = ax^2 + bx + c$ (Degree 2). Its graph is a parabola with vertex $\left(\frac{-b}{2a}, P\left(\frac{-b}{2a}\right)\right)$.

In this section, we will cover polynomial functions with degree 3 or higher.

All polynomial functions have graphs that are **smooth continuous curves**. A smooth curve is a curve that does not have a sharp curves and a continuous curve is a curve that does not have a break or a hole. (See the Figure)

Far-Left and Far-Right Behavior of a Polynomial with leading term $a_n x^n$.

If n is even, then if $1) a_n > 0$ the graph is up to left and up to right.
$2) a_n < 0$ the graph is down to left and down to right.

If n is odd, then if $1) a_n > 0$ the graph is down to left and up to right.
$2) a_n < 0$ the graph is up to left and down to right.

Example 1 *Examine the leading term to determine the far-left and the far-right behavior of the graph of each polynomial function:*

$1) P(x) = 2x^4 - 3x^3 - x^2 - 6x + 1$ $2) P(x) = -6x^3 + 10x^2 - 14x + 15$ $3) P(x) = \frac{1}{4}x^3 - x^2 - 6x - 4$ $4) P(x) = -5x^4 - 30x^3 + 2x^2 + 3x + 10.$

Turning Points are points where the function changes from an increasing function to an decreasing function, or vice versa. In general, the graph of a polynomial function of degree n has at most $n - 1$turning points.

The minimum value of a function f is the smallest range value of f. It is called the **absolute minimum**. The maximum value of a function f is the largest range value of f. It is called the **absolute maximum**.

Let f be a function defined on the open interval I, and let c be an element of I. Then

$f(c)$ is a **relative minimum** of f on I if $f(c) \leq f(x)$ for all x in I.

$f(c)$ is a **relative maximum** of f on I if $f(c) \geq f(x)$ for all x in I.

Example 2 *Identify the y-value of each of the points A, B, C, D, and E in the figure as a relative maximum, relative minimum, an absolute minimum or an absolute maximum.*

Example 3 *Let $S(x) = (2-x)^2 + (6-x)^2 + (7-x)^2$. Find the number x for which the value of S is a minimum. Now show that this number is the average of 2, 6, and 7.*

The Zero Location Theorem

Let $P(x)$ be a polynomial. If $a < b$, and if $P(a)$ and $P(b)$ have opposite signs, then there is at least one value c between a and b such that $P(c) = 0$.

Example 4 *Verify that* $P(x) = x^3 - 3x - 5$ *has a real zero between* 2 *and* 3.

Polynomials, Real Zeros, Graphs, and Factors $(x - c)$

If P is a polynomial and c is a real number, then all of the following statements are equivalent.

1) $(x - c)$ is a factor of P. 2)$x = c$ is a real solution of $P(x) = 0$. 3)$x = c$ is a real zero of P. 4) $(c, 0)$ is an x-intercept of the graph of $y = P(x)$.

Example 5 *Factor* $S(x) = x^3 - 2x^2 - 5x + 6$ *using the last theorem where its graph is the following figure.*

Even and Odd Powers of $(x - c)$

If c is a real number, and the polynomial $P(x)$ has $(x - c)$ as a factor exactly k times, then the graph of P will

intersect but not cross the x-axis at $(c, 0)$, provided that k is an even positive integer.

cross the x-axis at $(c, 0)$, provided that k is an odd positive integer.

Example 6 *Find the x-intercepts and determine where the graph of* $P(x) = (x - 3)^4 (x - 2)^3 (x + 1)^2 (x + 3)$ *will cross the x-axis and where the graph will intersect but not cross the x-axis and then graph it.*