1 Section 3.1 Polynomial Division and Synthetic Division

Polynomial Division

Dividing a polynomial by another polynomial is similar to the long division process used for diving positive integers.

Example 1 *Divide* $\left(x^2 + 9x + 12\right)$ *by* $(x + 4)$

Dividend $x^2 + 9x + 12$ **Divisor** $x + 4$ **Quotient** $x + 5$ **Remainder** -8

The dividend is equal to the product of the divisor and the quotient plus he remainder. $x^2 + 9x + 12 = (x + 4)(x + 5) - 8$.

The Division Algorithm for Polynomials

If $P(x)$ and $D(x)$ are polynomials such that $D(x) \neq 0$, then there exist unique polynomials $Q(x)$ and $R(x)$ such that $P(x) = D(x)Q(x) + R(x)$, where either $R(x) = 0$ or the degree of $R(x)$ is less than the degree of $D(x)$.

$\frac{P(x)}{Q(x)} = D(x) + \frac{R(x)}{Q(x)}$.

Example 2 *Perform the indicated division* 1) $\frac{x^4+x^3-11x^2+27x-11}{x^2-3x+4}$ 2) $\frac{x^4+3x^2-6x-10}{x^2+3x-5}$

Synthetic Division

We can divide a polynomial by a binomial of the form $x - c$, by a method called a synthetic division.

If we apply the long division for $\frac{3x^3-8x^2+7x+2}{x-2}$, and try to do the following steps, we will get the synthetic division.

1)Omit the variables.

2)Omit the repeated coefficients.

3)Change the sign of the divisor.

Example 3 *Use synthetic division to perform the indicated operation* $\frac{x^4-4x^3+10x+12}{x-3}$.

The Remainder Theorem

If a polynomial $P(x)$ is divided by $x - c$, then the remainder is $P(c)$.

Example 4 *Use the remainder theorem to evaluate* $P(x) = 3x^3 + 5x^2 - 11x + 3$ *when* $x = -3$ *and* $x = 2$.

Zero of a polynomial

If $P(x)$ is a polynomial and a is a number for which $P(a) = 0$, then a is a zero of $P(x)$.

The Factor Theorem

A polynomial $P(x)$ has a factor $(x - c)$ if and only if $P(c) = 0$. That is $(x - c)$ is a factor of $P(x)$ if and only if c is a zero of $P(x)$.

Example 5 *Determine whether* $(x + 2)$ *is a factor of* $P(x) = x^3 + 3x^2 + 4x + 4$.

Example 6 *Determine whether* $(x-2)$ *is a factor of* $P(x) = x^3 + 3x^2 + 4x + 4$.

$P(x) = x^3 + 3x^2 + 4x + 4 = (x+2)\left(x^2 + x + 2\right)$

Reduced Polynomials

$Q(x) = x^2 + x + 2$ is called a reduced polynomial or a depressed polynomial because it is one degree less than the degree of $P(x)$.

Example 7 *Verify that* $(x+4)$ *is a factor of* $P(x) = 3x^4 + 11x^3 - 6x^2 - 6x + 8$, *and write* $P(x)$ *as the product of* $(x+4)$ *and the reduced polynomial* $Q(x)$.

Example 8 *Use the factor theorem to prove that for any positive odd integer* n, $x^n - 1$ *has* $x - 1$ *as a factor.*

Example 9 *Find the remainder of* $5x^{48} + 6x^{10} - 5x + 7$ *divided by* $x - 1$.

Example 10 *Use synthetic division to show that* $(x-i)$ *is a factor of* $x^3 + 3x^2 + x - 3$.

Example 11 *Find the values of* k *so that when* $x^2 - 3x - 8$ *is divided by* $x + k$, *the remainder is equal to* -4.

Example 12 *If* $x^4 + 2x^3 - 2x - 2 = (x-1)(x+1)\,g(x) - 1$, *then find* $g(x)$.

Example 13 *If* $P(x) = x^{105} - x^{10} - 2x + 1$ *is divided by* $x - i$, *then find the remainder.*

If

$$\begin{array}{c|cccc} i & 1 & i & m & 2 \\ & & i & n & w \\ \hline & k & l & t & 2+i \end{array}$$

where $i = \sqrt{-1}$ is the synthetic division of some Polynomial $P(x)$ by $x - i$, then find the quotient.