1 Section 2.5 Properties of Graphs

Symmetry

The graph of an equation is symmetric with respect to

The y-axis if the replacement of x with -x leaves the equation unaltered.

The x-axis if the replacement of y with -y leaves the equation unaltered.

Example 1 *Determine whether the graph of the given equations has symmetry with respect to either the x- or the y-axis.* $1) x + y^2 = 4 \qquad 2) |x| + y = 4$

A graph is symmetric with respect to a point Q if for each point P on the graph, there is a point $P^{\backslash}$ on the graph such that Q is the midpoint of the line segment $PP^{\backslash}$.

The graph of an equation is symmetric with respect to the origin if the replacement of x with -x and of y with -y leaves the equation unaltered.

Example 2 *Determine whether the graph of each equation has symmetry with respect to the origin?* $1) xy = 4 \qquad 2) y = x^3 + 1 \qquad 3) |x| + |y| = 2$

Example 3 *Plot the image of the point* $Q(-4, 1)$ *with respect to the 1)y-axis 2)x-axis 3)origin.*

Even and Odd Functions

The function f is an even function if $f(-x) = f(x)$ for all x in the domain.

The function f is an odd function if $f(-x) = -f(x)$ for all x in the domain.

Example 4 *Determine whether each function is even, odd or neither.* $1) f(x) = 4x^5 + 5x \qquad 2) f(x) = |3x| \qquad 3) f(x) = x^3 + 2x^2$

Note that the graph of an even function is symmetric with respect to the y-axis and the graph of an odd function is symmetric with respect to the origin.

Translations of Graphs **Vertical Translations**

If f is a function and c is a positive constant, then the graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted up vertically c units.

The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted down c units.

Horizontal Translations

If f is a function and c is a positive constant, then the graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted left horizontally c units

The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted right c units

Example 5 *Graph each of the following using vertical and horizontal translations of the graph of* $f(x) = x^2$.

$1) g(x) = x^2 + 1 \qquad 2) h(x) = (x - 2)^2 \qquad 3) p(x) = (x - 2)^2 + 1$

Reflections of graphs

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis.

The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis.

Example 6 *Use reflections of the graph of* $f(x) = |x-2|+3$ *to graph* $1)g(x) = -(|x-2|+3)$ $\quad 2)h(x) = |-x+2|+3$

Shrinking and stretching of graphs
Vertical shrinking and stretching
If $0 < c < 1$, then the graph of $y = cf(x)$ is obtained by shrinking $y = f(x)$
If $c > 1$, then the graph of $y = cf(x)$ is obtained by stretching $y = f(x)$

Example 7 *Graph* $1)H(x) = \frac{1}{2}|x| + 2$ $\quad 2)G(x) = 3|x| + 2$

Horizontal shrinking and stretching
If $a > 0$ and the graph of $y = f(x)$ contains the point (x, y), then the graph of $y = f(ax)$ contains the point $(\frac{1}{a}x, y)$.
If $a > 1$, then the graph of $y = f(ax)$ is a horizontal shrinking of $y = f(x)$.
If $0 < a < 1$, then the graph of $y = f(ax)$ is a horizontal stretching of $y = f(x)$.

Example 8 *Use the graph of* $y = f(x)$ *to graph* $1)y = f(2x)$ $\quad 2)y = f(\frac{1}{3}x)$

Exercise 9 *Determine whether the graph of each equation has a symmetry with respect to the x-axis, the y-axis or the origin.*

$1)|xy| + |x|y = 1$ $\quad 2)|y| = \frac{|x+2|}{x^2}$ $\quad 3)|x| = |x-y|$ $\quad 4)y^2 = |x+1| - 3x^2$ $\quad 5)(xy)^2 - 2xy = 3$ $\quad 6)x^2 = |x-y^3|$ $\quad 7)y = \frac{x^2-3}{2x}$

Exercise 10 *Determine whether each function is even, odd or neither.*

$1)f(x) = x|x|$ $\quad 2)f(x) = 5$ $\quad 3)g(x) = \frac{1}{2}[f(x) + f(-x)]$ for any nonzero function f. $\quad 4)f(x) = \sqrt{3-x^2}$ $\quad 5)g(x) = \frac{x^3}{x^2+1}$ $\quad 6)h(x) = 2 + x + x^2$

Exercise 11 *Which translations should be applied to obtain the graph of the equation* $y = |x+3| - 2$ *from the graph of* $y = |x-1| + 3$

Exercise 12 *Which translations should be applied to obtain the graph of the equation* $y^2 - 2y - 4x - 7 = 0$ *from the graph of* $y^2 = 4x$.

Exercise 13 *What is the resulting function of shifting the graph of the equation* $x = y^2 + y$ *one unit to the left and two units upward*

Exercise 14 *What is the resulting function of shifting the graph of the equation* $y = \frac{x+2}{x-1}$ *one unit to the right and three units downward*

Exercise 15 *How can we obtain the graph of the equation* $g(x) = \sqrt{-x-1}+2$ *from the graph of* $f(x) = \sqrt{x}$

Exercise 16 *The graph of a function* $y = f(x)$ *is shown in the adjacent figure. Find the Domain and the Range of the function* $y = -f(x+1) + 2$.