

1 2.4 Quadratic functions

Definition 1 a quadratic function of x is a function that can be represented by an equation of the form $f(x) = ax^2 + bx + c$ where a , b , and c are real numbers and $a \neq 0$.

Example 2 Identify each function as linear or quadratic 1) $2x^2 - 3$. 2) $3 - 2x$.

The graph of $f(x) = ax^2 + bx + c$ is a parabola opens up if $a > 0$. and opens down if $a < 0$.

Example 3 Tell whether the parabola opens up or down: 1) $f(x) = x^2 - 3x + 4$. 2) $f(x) = 3x - 2x^2 + 4$.

Definition 4 The vertex of a parabola is the lowest point on a parabola that opens up or the highest point on a parabola that opens down.

The graph of $f(x) = ax^2 + bx + c$ is symmetric with respect to a vertical line through its vertex.

Symmetry with respect to a line

Definition 5 A graph is symmetric with respect to a line L if for each point P in the graph there is a point P' on the graph such that the line L is the perpendicular bisector of the line segment PP' . The line L is called the axis of symmetry.

$f(x) = ax^2 (a \neq 0)$ is a parabola with its vertex at the origin and the y-axis is its axis of symmetry.

Definition 6 Standard form of a quadratic functions

$f(x) = a(x - h)^2 + k$ is a parabola with vertex (h, k) . Opens up if $a > 0$ and down if $a < 0$. The vertical line $x = h$ is the axis of symmetry.

Example 7 Determine the vertex and axis of symmetry and whether its open up or down? 1) $f(x) = 3(x - 2)^2 - 4$. 2) $f(x) = 2(x + 4)^2 - 2$.

Example 8 Use the technique of completing the square to find the standard form of $f(x) = 2x^2 - 4x - 1$. Then sketch the graph.

Definition 9 Vertex of a parabola

The vertex of the graph of $f(x) = ax^2 + bx + c$ is $(-\frac{b}{2a}, f(-\frac{b}{2a}))$.

Example 10 Use the vertex formula to find the vertex and standard form of $f(x) = 2x^2 - 4x + 7$.

Definition 11 Range of $f(x) = ax^2 + bx + c$ and Maximum and Minimum

If $a > 0$, then the vertex (h, k) is the lowest point on the graph of $f(x) = a(x - h)^2 + k$, and the y-coordinate k of the vertex is the minimum value of the function f and the range is $\{y|y \geq k\}$.

If $a < 0$, then the vertex (h, k) is the highest point on the graph of $f(x) = a(x - h)^2 + k$, and the y-coordinate k of the vertex is the maximum value of the function f and the range is $\{y|y \leq k\}$.

Example 12 Find the maximum or minimum value of each quadratic function. State whether the value is maximum or minimum.

$$1)f(x) = \frac{1}{2}x^2 - 4x - 3. \quad 2)f(x) = -4x^2 + 8x - 10.$$

Example 13 Find the range of

$$1)f(x) = 2x^2 - 8x + 3. \quad 2)f(x) = -2x^2 - 6x - 1.$$

Example 14 Determine the value of x (if any) for which 1) $f(x) = 3$. 2) $f(x) = \frac{7}{2}$. 3) $f(x) = 5$. where $f(x) = -2x^2 - 6x - 1$.

Example 15 Find the x - and y - intercepts (if any) of 1) $f(x) = -x^2 + 4x$. 2) $f(x) = 2x^2 + 3x + 4$.

Applications

Example 16 The height of a ball thrown upward at an initial speed of 64 ft. per sec from a platform 50 ft. above the ground is given by the function $s(t) = -16t^2 + 64t + 50$, where $s(t)$ is measured in feet above the ground level and t is the time in seconds.

- Determine the time it takes the ball to attain its maximum height. (ans: 2s)
- Determine the maximum height the ball attains. (ans: 114 ft.)
- Determine the time it takes the ball to hit the ground. (ans: 4.67 s)

Example 17 Find the quadratic function of x whose graph has a maximum at $(-3, 2)$ and passes through $(0, -5)$

Example 18 Find two numbers whose difference is 12 and whose product is a minimum.

Example 19 Find the largest possible value of $\sqrt{-3x^2 - 2x + 1}$. (ans: $\frac{\sqrt{3}}{4}$)

Example 20 If $f(x) = -x^2 - 16$, then find the interval where f is decreasing. (ans: $[0, \infty)$)

Example 21 If one of the x -intercepts of an open downward parabola with vertex at $(-1, 8)$ is equal to -3 , then find the other x -intercept. (ans: 0)

Example 22 Given the function $f(x) = x^2 + 4x + 2$ with domain $[-3, 0]$, then find the minimum and maximum values of $f(x)$. (ans: $-2, 2$)