

## 1 2.3 Linear functions

**Definition 1**  $f(x) = mx + b, m \neq 0$ . is a linear function of  $x$  and this is a nonvertical linear function.

In the figure, the lines intersect at the same point but differs in the steepness (the slope).

**Definition 2** The slope  $m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$  where the line passes through the points  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ .

The numerator  $y_2 - y_1$  is the vertical rise from  $P_1$  to  $P_2$  and the denominator  $x_2 - x_1$  is the horizontal run from  $P_1$  to  $P_2$ .

**Example 3** Graph the line with slope  $\frac{3}{2}$  and passes through the point  $(1, 2)$ .

Lines that have a positive slope slant upward from left to right and lines that have a negative slope slant downward from left to right.

**Example 4** Find the slope of the line passing through the points whose coordinates are: 1)  $(3, 4)$  and  $(5, 10)$ . (ans:  $m=3$ ) 2)  $(-4, -3)$  and  $(1, -2)$ . (ans:  $m=\frac{1}{5}$ )

### Special cases:

1- Vertical line passes through the point  $(a, 0)$ . Slope: Equation:  
Is it a function or not?

2- Horizontal line passes through the point  $(0, b)$ . Slope: Equation:  
Is it a function or not?

### Slope-Intercept Form

The graph of  $f(x) = mx + b$  is a line with slope  $m$  and y-intercept  $(0, b)$ .

**Example 5** Graph the following lines 1)  $f(x) = 2x - 1$  2)  $f(x) = -3x + 4$

**Example 6** Find the equation of a line with y-intercept  $(0, 3)$  and slope  $\frac{3}{4}$ .

### Point-slope Form

The graph of  $y - y_1 = m(x - x_1)$  is a line that has slope  $m$  and passes through the point  $(x_1, y_1)$ .

**Example 7** Find the equation of a line with slope  $\frac{2}{3}$  and passes through the point  $(-3, 4)$ .

**Example 8** Find the equation of a line that passes through the points  $(3, 1)$  and  $(-1, 4)$ .

**Example 9** Find the equation of a line with x-intercept  $(3, 0)$  and passes through the point  $(2, 1)$ .

**General form of the equation of a line:**  $Ax + By + C = 0, A, B \neq 0$ .

**Example 10** Write the equation of the line in its general form.  $y = -3x + 1$ .

**Example 11** Determine the slope,  $y$ -intercept and  $x$ -intercept of the following lines: 1)  $f(x) = \frac{-1}{4}x + 3$ . 2)  $3x + 2y = 6$ .

**Example 12** Find the value of  $x$  in the domain of  $f(x) = 5x + 12$ , for which  $f(x) = 2$ .

**Theorem 13 Real solutions and  $x$ -intercept theorem:** For every function  $f$ , the real number  $c$  is a solution of  $f(x) = 0$  if and only if  $(c, 0)$  is an  $x$ -intercept of the graph of  $y = f(x)$ .

**Example 14** Let  $f(x) = 2x + 4$ . Find the real solution of  $f(x) = 0$  and then graph  $y = f(x)$ . Compare the solution of  $f(x) = 0$  with the  $x$ -intercept of the graph of  $f$ .

**Example 15** Let  $f_1(x) = \frac{3}{2}x + 6$  and  $f_2(x) = -2x + 1$ . Find the value of  $x$  for which  $f_1(x) = f_2(x)$ .

**Parallel and perpendicular lines:**  $l_1: f_1(x) = m_1x + b_1$  &  $l_2: f_2(x) = m_2x + b_2$

**Definition 16**  $l_1$  and  $l_2$  are parallel if  $m_1 = m_2$ .

**Definition 17**  $l_1$  and  $l_2$  are perpendicular if  $m_1m_2 = -1$ .

**Example 18** Find the equation of a line that passes through  $(1, 3)$  and parallel to the line  $3x + 4y = -24$ .

**Example 19** Find the equation of a line that passes through  $(1, 2)$  and perpendicular to the line  $2x - y = 7$ .

**Example 20** Find a point  $P(x, y)$  on the graph of the equation  $y = x^2$  such that the slope of the line through the point  $(3, 9)$  and  $P$  is  $\frac{5}{12}$ .

**Example 21** Let  $f$  be a linear function such that  $f(9) = 0$  and the graph of  $f$  is parallel to the line  $x - 3y - 4 = 0$ , then find  $f(3)$ . (ans: -2)

**Example 22** Find the value of  $k$  where the line through the points  $(4, -1)$  and  $(k, 2)$  is perpendicular to the line  $2x - 5y = 1$ . (ans:  $\frac{14}{5}$ )

**Example 23** Find the  $x$ -intercept and the  $y$ -intercept of the line passing through the points  $(-2, -1)$  and  $(1, 3)$ . (ans:  $-\frac{5}{4}, \frac{5}{3}$ ).

**Example 24** If  $f(5) = -2$ ,  $f(1) = 0$ , and  $f$  is a linear function, then find  $f(-4)$ . (ans:  $\frac{5}{2}$ )

**Example 25** If the points  $(k, -8), (-1, 7)$  and  $(2, -2)$  lie on the same straight line, then find the value of  $k$ .

**Example 26** Let  $f$  be a linear function such that  $f(9) = 0$  and the graph of  $f$  is parallel to the line  $x - 3y - 4 = 0$ , then find the value of  $f(3)$  (ans: -2)

**Example 27** Find the  $y$ -intercept of the line passing through  $(2, -5)$  and perpendicular to the line  $3x + 2y = 5$ . (ans:  $-\frac{19}{3}$ )