1 Section 2.2 Introduction to Functions

The correspondence between two sets is often defined by a table, an equation, or a graph, each of which can be viewed as a set of ordered pairs. Any set of ordered pairs is called a *relation*. A **function** is a set of ordered pairs in which no two ordered pairs have the same first coordinate and different second coordinate. Note that *every function is a relation, but not every relation is a function.*

Example 1 *Does the set* $\{(0,0),(1,0),(2,0),(3,0),(4,0)\}$ *define a function?*

Example 2 *Does the set* $\{(3,4),(6,7),(3,5)\}$ *define a function?*

Definition 3 *The* ***Domain*** *of a function is the set of all first coordinates of the ordered pairs.*

Definition 4 *The* ***Range*** *of a function is the set of all second coordinates of the ordered pairs.*

If a function is defined by an equation, then the variable that represents elements of the domain is called the *independent variable* but the variable that represents elements of the range is called the *dependent variable.*

Example 5 *Consider the relation* $\{(0,1),(2,3),(4,5),(6,7)\}$ *1- Is the relation a function? 2- Find the Domain and the range of the relation.*

Functional Notation: If x is an element of the domain of a function f, then $f(x)$, which is read "f of x " or " the value of f at x ", is the element in the range that corresponds with the domain element x.

Note that "f " is the name of the function, whereas "$f(x)$" is the value of the function at x.

Example 6 *If* $f=\{(0,1),(2,3),(4,5),(6,7)\}$, *What is* $f(2)$*?*

Example 7 *Let* $f(x)=2x^2-2$, *and evaluate 1)*$f(-1)$. 2)$f(2a)$ 3)$2f(a)$ 4)$f(b-3)$ 5)$f(b)-f(3)$

Piecewise Defined Functions: (more than one expression).

Example 8 *If* $f(x)=\begin{cases} 2x, \text{ if } x<1 \\ x^2, \text{ if } 1\le x<5 \\ 4, \text{ if } x\ge 5 \end{cases}$ *, then find the value of* 1)$f(-4)$ 2)$f(1)$ 3)$f(3)+f(5)$ 4)$f(10)$ 5)$f(k+4)$, *where* $k\ge 1$.

Identifying functions:

Example 9 *Which relations define* y *as a function of* x? $1)\{(3,4),(6,7),(3,6)\}$ $2)x+3y=-6.$ $3)x^2+y^2=16$ $4)x^2+y=3$

$5-$ *The correspondence between the x-values and the y-values in the figure*

Domain of a function: Unless otherwise stated, the domain of a function is the set of all real numbers for which the function makes sense and yields real numbers.

Example 10 *Determine the Domain of the following functions:* $1)f(x)=x+3$ $2)f(x)=\sqrt{x-4}$ $3)f(x)=\frac{x+3}{x+2}$ $4)f(x)=\frac{\sqrt{x-2}}{x-3}$

$5)V(r)=\frac{4}{3}r^3$, *where* $V(r)$ *is the volume of a sphere whose radius is* r *units.*

Example 11 *Find the Domain and the Range of the following functions:* $1)f(x)=x+3$ $2)f(x)=x^2-4$ $3)f(x)=\sqrt{x-4}$ $4)f(x)=-\sqrt{x-2}+1$ $5)f(x)=\sqrt[3]{x-3}$

The Vertical Line Test for Functions:

A graph is the graph of a function if and only if no vertical line intersects the graph at more than one point.

Example 12 *Which of the following graphs are graphs of a functions?*

Increasing, Decreasing, and Constant Functions

If a and b are elements of an interval I that is a subset of the domain of a function f, then

f is increasing on I if $f(a)<f(b)$ whenever $a<b$, f is decreasing on I if $f(a)>f(b)$ whenever $a<b$, and f is constant on I if $f(a)=f(b)$ whenever $a<b$.

Example 13 *At what intervals is the following graph 1-Increasing. 2-Decreasing 3-Constant.*

One-to-One Function: Given any y, there is only one x that can be paired with that given y. If every Horizontal line intersects the graph of a function at most one, then the graph is a graph of a one-to-one function.

Example 14 *Which of the following is a one-to-one function? 1)The previous graph. 2)*$f(x)=2x$ *3)*$f(x)=x^{2.}$

Greatest Integer Function (Step Function)

Definition 15 $[x]$ *or* $int(x)$ *is defined to be the greatest integer less than or equal to* x.

Example 16 $int(-3)=-3$ $int(-7.4)=-8$ $int(\pi)=3.$

Domain: All real numbers Range: Integers

Exercise 17 *Graph 1)*$f(x)=[x]$ *2)* $f(x)=[\frac{1}{2}x]$ *3)*$f(x)=[x]+2$ *4)*$f(x)=[-x]$

Exercise 18 *Find the x- and y-intercepts of* $f(x)=[2x-\frac{1}{2}]$.