1 Section 2.1 A Two Dimensional Coordinate System and Graphs

Each point on a coordinate axis is associated with a number called its *coordinate*. Each point on the xy-plane (Coordinate Plane) is associated with an order pair of numbers called coordinates of the point. The ordered pairs denoted by (a, b) where a is the *x-coordinate* and b is the *y-coordinate*. The horizontal coordinate axis is called the *x-axis* and the vertical coordinate axis is called the *y-axis*. They intersect at the point $(0, 0)$ which is called *the origin*. There are four regions formed by the axes and they are called the *Quadrants* and they are numbered counterclockwise.

The *Cartesian coordinate System* is the name of this two-dimensional coordinate system.

Example 1 *Plot the points* $(1, 3), (3, 1), (-2, 3), (0, -2)$.

Equality of two ordered pairs: The ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.

Example 2 *If* $(3, y) = (x, -2)$*, then find the value of* x *and* y.

Distance and Midpoint Formulas

Definition 3 *The distance between the points* $P_1(x_1, y_1)$ *and* $P_2(x_2, y_2)$ *is* $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Definition 4 *The Midpoint* M *of the line segment from* $P_1(x_1, y_1)$ *and* $P_2(x_2, y_2)$ *is given by* $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Example 5 *Find the distance between the points* $(3, 4)$ *and* $(7, -2)$.

Example 6 *Find the midpoint of the line segment that connects the points* $(3, 4)$ *and* $(7, -2)$.

Example 7 *Find the other endpoint of the line segment that has the endpoint* $(4, -6)$ *and the midpoint* $(-2, 11)$.

Graph of an Equation

The graph of an equation in the two variables x and y is the set of all points where coordinates satisfy the equation. (Generally, there are an infinite number of solutions of an equation in two variables).

Example 8 *Find ordered pair solutions to* $y = |x| + x$ *corresponding to* $x = -2, -1, 0, 1, 2$.

Example 9 *Graph* $y = 2x + 1$ *(Graph by plotting points)*

Example 10 *Graph* $1) x^2 - y = 4 \qquad 2) y = |x + 3| \qquad 3) y^2 = x$

Intercepts: Any point that has an x- or a y- coordinate of zero is called an intercept of the graph of an equation, because at these points the graph intersects the x- or the y-axis. (*x-intercept* $y = 0$ *y-intercept* $x = 0$).

Example 11 *Identify each of the following as an x-intercept , a y-intercept, both , or neither:* 1) $(0, -5)$ 2) $(2, 0)$ 3)$(1, 1)$ $4 - (0, 0)$

the Equation of the x-axis: the Equation of the y-axis:

Example 12 *Find the x- and y-intercepts of the following graphs and use them and additional points to graph the equation:*

1)$y = x^2 + 3x - 4$ 2)$y = |x + 3|$ 3)$|x| + |y| = 2$

Circles, their equations and their graphs

Circle is the set of points in a plane that are a fixed distance from a specified point. The distance is the *Radius* of the circle, and the specified point is the *Center* of the circle.

The *standard form* of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

Example 13 *What each of these variables represent and state whether they remain a letter or replaced by a number?*

Example 14 *Find the standard form of the equation of a circle that has the following information:*

1)Center $(0, 0)$, and radius 3.
2)Center $(1, 2)$, and radius 3.
3)Center $(0, 1)$ and passing through the point $(1, 7)$.
4)A diameter with endpoints $(7, -2)$ and $(-3, 5)$.
5)Center $(-2, 3)$ and tangent to the y-axis.
6)Tangent to both axes, center in the third quadrant, and radius of $\sqrt{5}$.

If we rewrite $(x + 4)^2 + (y + 2)^2 = 3^2$, then we will get the following $x^2 + y^2 + 8x + 4y - 5 = 0$ and this is called the *general form* of the equation of the circle.

$x^2 + y^2 + Ax + By + C = 0$ (General Form) $\overset{\text{Completing the Square}}{\longrightarrow\longrightarrow\longrightarrow}$ $(x - h)^2 + (y - k)^2 = s$ (Standard Form)

If $s > 0$, then the equation represents a circle with center (h, k) and radius r. If $s = 0$, then the equation represents the point (h, k) . If $s < 0$, then the equation represents NO GRAPH.

Example 15 *What does each of the following equations represent and if it is a circle then find its center and radius?*

1)$x^2 + y^2 = 25$
2)$x^2 + y^2 + 6x - 4y + 12 = 0$
3)$x^2 + y^2 + 6x - 4y + 13 = 0$
4)$x^2 + y^2 + 6x - 4y + 15 = 0$
5)$4x^2 + 4y^2 + 4x - 63 = 0$