

1 1.4 Other Types of Equations

Solving higher-Degree Equations by Factoring

Example 1 Find the solution set of the following equations: 1) $x^3 - 36x = 0$ 2) $x^4 + 3x^3 - 8x - 24 = 0$

Solving equations by Using the **Power Principle** (Equations that involve radical expressions)

The Power Principle Every solution of $P = Q$ is a solution of $P^n = Q^n$

Example 2 Find the solution set of the following equations: 1) $\sqrt{2x-3} = 7$ 2) $x + 2\sqrt{x-1} = 9$ 3) $x - 2\sqrt{x-2} = x + 8$ 4) $\sqrt{x-2} - \sqrt{x} = -2$

Note that we need to check after we find the solutions.

(Equations involve fractional exponents: We need to raise each side to a reciprocal power)

Example 3 Find the solution set of the following equation: 1) $x^{\frac{1}{3}} = 4$ 2) $(x^2 - 15x - 19)^{\frac{3}{4}} = 27$

Solving equations that are quadratic in form

Example 4 Which of the following are quadratic in form: 1) $x^4 + 4x^2 - 12 = 0$ 2) $x^3 + 4x - 12 = 0$ 3) $x + 4x^{\frac{1}{2}} - 12 = 0$ 4) $x^9 + 4x^3 - 12 = 0$ 5) $x^3 + 4x^{\frac{3}{2}} - 12 = 0$

Example 5 Find the solution set of the following equations: 1) $x^8 - 17x^4 + 16 = 0$ 2) $x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 3 = 0$ 3) $4x^{\frac{2}{3}} - 4x^{\frac{1}{3}} - 3 = 0$

Example 6 Find the solution set of the following equations: 1) $\sqrt[3]{7x-3} = \sqrt[3]{2x+7}$ 2) $\sqrt{3\sqrt{5x+16}} = \sqrt{5x-2}$ 3) $10\left(\frac{x-2}{x}\right)^2 + 9\left(\frac{x-2}{x}\right) - 9 = 0$ 4) $3 - 7\sqrt[3]{x-1} + 2(x-1)^{\frac{2}{3}} = 0$ 5) $|2x-1|^3 - 5|2x-1|^2 + 4|2x-1| = 0$

$$6) (x-2)^{\frac{2}{3}} - (8x-16)^{\frac{1}{3}} = 3$$