1 1.3* Complex Numbers

If we solve the equation $x^2 = -1$, then we will get that $x = \pm\sqrt{-1}$. Since $\sqrt{-1}$ is not a real number, we will define $i = \sqrt{-1}$ and we will call it an **imaginary number** where $i^2 = -1$.

Definition of $\sqrt{-a}$

For any positive real number a, $\sqrt{-a} = i\sqrt{a}$.

Example 1 *Write $\sqrt{-4}$ in terms of i, where $i = \sqrt{-1}$.*

Definition of a Complex Number

If a and b are real numbers and i is the imaginary unit, then $a + bi$ is called a **complex number**. The real number a is called the **real part** and the real number b is called the **imaginary part** of the complex number.

The real numbers are a subset of the complex number.(note that $a + bi = a$ with $b = 0$)

$0 + bi = bi$ is an imaginary number.

The **standard form** of a complex number is $a + bi$.

Example 2 *Write the following in its standard form* $1)3+\sqrt{-4}$ $\quad 2)\sqrt{-37}-3$

Addition and Subtraction

$(a + bi) + (c + di) = (a + c) + (b + d)\, i$

$(a + bi) - (c + di) = (a + c) - (b + d)\, i$

Example 3 *Perform the indicated operation:* $1)\,(4 + 2i) + (3 + 7i)$ $\quad 2)i - (3 - 4i)$

Multiplication

$(a + bi)\,(c + di) = (ac - bd) + (ad + bc)\, i$

Example 4 *Perform the indicated operation:* $(3 + 5i)\,(2 - 4i)$

Caution To compute $\sqrt{a}\sqrt{b}$ when both are negative numbers, write each radical in terms of i before multiplying.

Example 5 *Simplify and write the result in standard form:* $1)\sqrt{-20}\sqrt{-5}$
$2)\,\left(2 + 5\sqrt{-3}\right)\left(1 - 2\sqrt{-3}\right)$

The **conjugate** of the complex number $z = a + bi$ is $\overline{z} = a - bi$.

Example 6 *Simplify and write the result in standard form:* $1)\overline{3 + 2i}$ $\quad 2)\overline{7 - 11i}$

The product of a complex number and its conjugate is a real number. $((a + bi)\,(a - bi) = a^2 + b^2)$

Example 7 *Simplify and write the result in standard form:* $1)\,(3 + 2i)\,(3 - 2i)$ $\quad 2)\,(3 - 5i)^2$

Division (Multiply the numerator and the denominator by the conjugate of the denominator)

Example 8 *Simplify and write the result in standard form:* $\frac{3+2i}{5-i}$

Powers

$i^1 = i$	$i^5 = i$	$i^9 =$
$i^2 = -1$	$i^6 = -1$	$i^{10} =$
$i^3 = -i$	$i^7 = -i$	$i^{11} =$
$i^4 = 1$	$i^8 = 1$	$i^{12} =$

Example 9 *Find:*1)i^4 2)i^{4n} 3)i^{25}

If n is a positive number, then $i^n = i^r$, where r is the reminder of the division of n by 4.

Example 10 *Find:* 1)i^{543} 2) $(-i)^{47}$ 3)i^{-45}

The **absolute value** of the complex number is defined by $|a+bi| = \sqrt{a^2+b^2}$.

Example 11 *Find:* 1)$|2-5i|$ 2)$|-3i|$

Example 12 *If* $z = (5-3i)(-2-4i) + \sqrt{-1}\sqrt{-4}$, *then find the conjugate* $\overline{z}$.

Example 13 *Write the complex number* $\frac{4+i}{3+i} - \frac{1}{3-i}$ *in standard form.*

Example 14 *If* $z = \left(\frac{i}{1-i}\right)^2$, *then find* $z + \overline{z}$.

Example 15 *Simplify:* 1)$i + i^2 + i^3 + \cdots + i^{24}$. 2)$i^{49} + i^{50} + i^{51}$.

Example 16 *Let* $Z_1 = x - 4i$, $Z_2 = 5 - 2yi$. *If* $Z_2 = 3Z_1 - 5$, *then find the real numbers* x *and* y.

Example 17 *If* $A + Bi = \frac{\sqrt[3]{-125} + i^{11} - \sqrt{-4}\sqrt{-1}}{i-6}$, *then find the real numbers* A *and* B.

Example 18 *If* $z = \frac{i^{45}}{-2+i^{27}}$, *then find the real part of* z.