1 1.3 Quadratic Equation

A *quadratic equation* in x is an equation that can be written in the standard quadratic form $ax^2 + bx + c = 0, a \neq 0$.

Methods to solve Quadratic Equations

1)**Zero Factor Property** $AB = 0$ if and only if $A = 0$ or $B = 0$.

Example 1 *Find the solution set of the equations:* 1)$4x^2 - 2 = 7x$ 2)$36x^2 - 12x + 1 = 0$

Example 2 *Find the equation whose roots are:* 1)$3, -4$ 2)5 *as a double root*

2)**Taking Square Root** If $A^2 = B$, then $A = \pm\sqrt{B}$

Example 3 *Find the solution set of the equations:* 1)$(x+8)^2 = 81$ 2)$(x+2)^2 + 36 = 0$

3)**Completing the square** and that means adding to a binomial of the form $x^2 + bx$ a constant that makes that binomial a perfect-square trinomial. We will add the square of half the coefficient of x. Note that this method can be used to solve any quadratic equation.

Example 4 *What is the constant needed to complete the square of* $x^2 + 8x$

Example 5 *Find the solution set of the following equations:* 1)$x^2 - 6x = -13$ 2)$x^2 + 2 = x$ 3)$3x^2 + 18x - 4 = 0$

4)**Using the Quadratic Formula** If $ax^2+bx+c = 0$, then $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Example 6 *Find the solution set of the following equations:* 1)$4x^2 - 24x + 5 = 0$ 2)$x^2 + 7 = 5x$ 3)$x^2 + 10x + 25 = 0$

The discriminant

Theorem 7 *The quadratic equation $ax^2+bx+c = 0$, with real coefficients and $a \neq 0$, has discriminant $b^2 - 4ac$.*

If $b^2 - 4ac > 0$, then the quadratic equation has two distinct real roots
If $b^2 - 4ac = 0$, then the quadratic equation has real root that is a double root.
If $b^2 - 4ac < 0$, then the quadratic equation has two distinct complex roots that are not real. These roots are conjugate of each other.

Example 8 *Classify the roots of each quadratic equation as real numbers or non real complex numbers:* 1)$3x^2 + 4x - 5 = 0$ 2)$2x^2 - 4x + 5 = 0$ 3)$8x^2 - 8x + 2 = 0$

Theorem 9 *Sum and Product of the roots Theorem*

$r_1$and r_2 are roots of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$, if and only if $r_1 + r_2 = \frac{-b}{a}$ and $r_1 r_2 = \frac{c}{a}$

Example 10 *Determine whether the given numbers are roots of the quadratic equation:* $1)x^2 + 4x - 21 = 0, \qquad -7, 3 \qquad 2)2x^2 - 7x - 30 = 0 \qquad -3, 6$

Example 11 *Find the sum and the product of the roots of the equation* $2x^2 - 7x - 30 = 0$

Exercise 12 *If the product of the solutions of* $kx^2 - 4x + (2k - 1) = 0$ *is 3, then find k.*

Exercise 13 *Find all real values of k such that the equation* $x^2 + k^2 = 2(k+1)x$ *has exactly one real solution (two equal real solutions)*

Exercise 14 *If one solution of the equation* $kx^2 - 17x + 33 = 0$ *is 3, then find k and the other solution.*

Exercise 15 *If m and n are the solutions of the equation* $2x^2 - 2x + 1 = 0$*, then find the equation whose solutions are 3m and 3n.*

Exercise 16 *If the sum of the two roots of a quadratic equation is* $\frac{7}{2}$ *and the product of the two roots is -15, then find the quadratic equation.*

Exercise 17 *Solve for y in the equation* $3x^2 + xy + 4y^2 - 2 = 0$

Exercise 18 *If the sum of the squares of three consecutive positive integers a, b, and c is 149, then find these three numbers*