

1 Section 1.1 Linear Equations

An **equation** is a statement about the equality of two expressions. For example, $2x + 1 = 7$ is an equation and it is true for $x = 3$ but false for any number except 3. The number 3 is said to **satisfy** the equation $2x + 1 = 7$.

Solve an equation means find all values of the variable that satisfy the equation. **Solutions or roots** of the equation are the values that satisfy the equation. **Equivalent equations** are equations that have exactly the same solution(s).

Procedure that Produce Equivalent Equations

1) Simplification of an expression on either side

Example 1 $2x + 3 + 5x = -11$ and $7x + 3 = -11$ are equivalent equations

2) Addition or subtraction of the same quantity on both sides.

Example 2 $3x - 7 = 2$ and $3x = 9$ are equivalent equations.

3) Multiplication or division by the same nonzero quantity on both sides.

Example 3 $\frac{5}{6}x = 10$ and $x = 12$ are equivalent equations.

Linear Equation in x is an equation that can be written in the form $ax + b = 0$ where a and b are real numbers and $a \neq 0$.

Example 4 Solve the following equations: 1) $\frac{3}{5}x - 16 = -1$ 2) $\frac{3}{4}x + 2 - \frac{x}{2} = -3$ 3) $(x - 2)(2x + 3) = 2x(x - 1)$

Contradictions, Conditional Equations and Identities

An equation that has no solutions is called a **contradiction**.

Example 5 $x = x + 1$ is a contradiction.

An equation that is true for some values but not for other values is called a **conditional equation**.

Example 6 $x + 2 = 8$ is a conditional equation.

An equation that is true for every real number for which all terms of the equation are defined is called an **identity**.

Example 7 $x + x = 2x$ is an identity.

Example 8 Which one of the following statements is TRUE?

1) $(x + 4)^2 = x^2 + 16$ is an identity. 2) $x - 3 = 0$ and $x^2 = 9$ are equivalent equations. 3) $x(5 + x) = x^2 + 5(x + 1)$ is a contradiction. 4) $\frac{6x+1}{3} = 2x + \frac{1}{3}$ is a conditional equation. 5) $(x - 3)(x + 4) = x^2 - x - 12$ is an identity.

Example 9 Verify the identity $\frac{3(x^3-8)}{x-2} = 3x^2 + 6x + 12, x \neq 2$.

Solve Equations that Have Restrictions

If each side of an equations is multiplied by an expression that involves a variable, then we restrict the variable so that the expression is not equal to zero.

Example 10 Find the solution set of the following equations: 1) $\frac{x}{x-3} = \frac{9}{x-3} - 5$ 2) $1 + \frac{x}{x-5} = \frac{5}{x-5}$ 3) $\frac{8}{2m+1} - \frac{1}{m-2} = \frac{5}{2m+1}$

Absolute Value Equations: For any variable expression E and any non-negative real number k , $|E| = k$ if and only if $E = k$ or $E = -k$.

Example 11 Find the solution set of the following equations: 1) $|2x - 5| = 21$ 2) $|2x + 5| = 0$ 3) $|2x + 5| = -8$ 4) $|x - 3| = -(x - 3)$ 5) $2|x + 3| + 4 = 34$

2 Section 1.2 Formulas

Solve a formula for a specified variable in terms of the other variable(s).

Example 12 Solve $2l + 2w = p$ for l .

Example 13 Solve $xy - z = yz$ for y .

Example 14 Solve $z = y\left(1 + \frac{m}{x}\right)$ for x .