1 Section 10.3 The Inverse of a Matrix

Multiplicative Inverse of a Matrix

If A is a square matrix of order n, hen the inverse of matrix A, denoted by A^{-1}, has the property that $A \cdot A^{-1} = A^{-1} \cdot A = I_n$ where I_n is the identity matrix of order n.

Note that not all square matrices has a multiplicative inverse.

A procedure for finding the inverse uses elementary row operations. To the matrix A we will merge the identity matrix I to the right of A and denote this new matrix by $[A; I]$. Now, we will use elementary row operations to produce $[I; A^{-1}]$.

Example 1 *Find* A^{-1} *if* $A = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$.

Example 2 *Find the inverse of the matrix* $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 6 \\ -1 & -1 & 2 \end{bmatrix}$.

A **singular matrix** is a matrix that does not have a multiplicative inverse. A matrix that has a multiplicative inverse is a **non singular matrix**.

If there are all zeros in a row of the original matrix (after we apply the elementary row operations), then the matrix does not have an inverse.

Example 3 *Show that the matrix* $\begin{bmatrix} 1 & -6 & 4 \\ 3 & 4 & 2 \\ 5 & 3 & 5 \end{bmatrix}$ *is a singular matrix.*

Systems of equations can be solved by finding the inverse of the coefficient matrix in the following steps:

1) Write the linear system as a matrix equation in the form $AX = B$. 2)The solution is $X = A^{-1}B$.

Example 4 *Find the solution set of the following system of equations by using the inverse of the coefficient matrix:*

1) $\begin{cases} 3x_1 + 4x_2 = -1 \\ 3x_1 + 5x_2 = 1 \end{cases}$ 2) $\begin{cases} x_1 + \quad 7x_3 = 20 \\ 2x_1 + x_2 - x_3 = -3 \\ 7x_1 + 3x_2 + x_3 = 2 \end{cases}$

Example 5 *True or False: 1) If* $AB = O$, *then* $A = O$ *or* $B = O$. ($A = \begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$ *and* $B = \begin{bmatrix} -3 & 15 \\ -2 & 10 \end{bmatrix}$)

2) If A has an inverse and $AB = O$, then $B = O$.

3)If $AB = AC$, then $B = C$. ($A = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 4 & 7 \\ 3 & 11 \end{bmatrix}$)

4)If A has an inverse and $AB = AC$, then $B = C$.

Example 6 *Show that if* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *and* $ad - bc \neq 0$, *then* $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Example 7 *Find the inverse (if possible) of* 1) $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$ 2) $\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix}$.

Example 8 *Show that* $(AB)^{-1} = B^{-1}A^{-1}$.