DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics Term o41

QUIZ # 1

Name ____________________ ID # __________ Sec # _____

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In the following $\underline{r}$ represent the position vector, $\underline{a} = \langle a_1, a_2, a_3 \rangle$ is a constant vector, $\underline{F} = \langle F_1, F_2, F_3 \rangle$ is a vector field and *f(x,y,z)* is a scalar function. **Show** the following results.

Q1) $\underline{\nabla}.(\underline{\nabla} \times \underline{F}) = 0$. **We need to first find** $\underline{\nabla} \times \underline{F}$

$\underline{\nabla} \times \underline{F} =$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \underline{i}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - \underline{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + \underline{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

Hence,

$\underline{\nabla}.(\underline{\nabla} \times \underline{F}) = \frac{\partial}{\partial x}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) = 0$ (you may check by expanding).

Q2) $\underline{\nabla}.\{(\underline{r}.\underline{r})\underline{a}\} = 2(\underline{r}.\underline{a})$.

$\underline{r}.\underline{r} = \langle x, y, z \rangle . \langle x, y, z \rangle = x^2 + y^2 + z^2$

$(\underline{r}.\underline{r})\underline{a} = \langle (x^2 + y^2 + z^2)a_1, (x^2 + y^2 + z^2)a_2, (x^2 + y^2 + z^2)a_3 \rangle$

Hence $\underline{\nabla}.\{(\underline{r}.\underline{r})\underline{a}\} = \frac{\partial}{\partial x}(x^2 + y^2 + z^2)a_1 + \frac{\partial}{\partial y}(x^2 + y^2 + z^2)a_2 + \frac{\partial}{\partial z}(x^2 + y^2 + z^2)a_3$

$= 2(xa_1 + ya_2 + za_3) = 2(\underline{r}.\underline{a})$