Solution

DEPARTMENT OF MATHEMATICAL SCIENCES

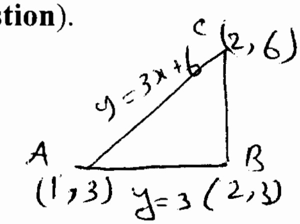
MATH 301 Methods of Applied Mathematics Term 061

QUIZ # 2(a)

Name ____________________ ID # __________ Section # ____

Q1) In the following question, use *Green's theorem* to write R.H.S of the given line integral as a double integral, showing correct integral limits (**Do not evaluate integrals in this question**).

(a) $\oint_C (x^2+y^2)dx + (x^2-y^2)dy = \int_1^2 \int_3^{3x+6} 2(x-y)\,dy\,dx$

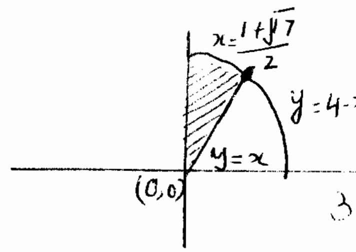


where C is triangle counterclockwise with vertices A(1,3), B(2,3) and C(2,6).

Slope of AB $= \frac{6-3}{2-1} = 3$, Equation: $y - 3 = 3(x-1) \Rightarrow y = 3x + 6$

(b) $\oint_C 4y\sin^2 x\,dx - 5x\cos^2 y\,dy = \int_0^{\frac{1+\sqrt{17}}{2}} \int_x^{4-x^2} (-5\cos^2 y - 4\sin^2 y)\,dy\,dx$

Where C is closed counter clockwise by $y = 4 - x^2, y - axis\ and\ y = x$.



R: $y = x$ to $y = 4 - x^2$

Solve these to get point of intersection $x = \frac{1+\sqrt{17}}{2}$

$x = 0$ to $\frac{1+\sqrt{17}}{2}$.

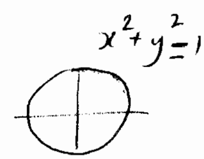
3

Q2) Evaluate the integral using **Green's theorem** $\oint_C -y\,dx + x\,dy$

Where C is $x^2+y^2=1$.

$P = -y, \quad Q = x \qquad \frac{\partial P}{\partial y} = -1, \quad \frac{\partial Q}{\partial x} = 1$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2.$



$\oint_C -y\,dx + x\,dy = \iint_R 2\,dA$

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Use polar coordinates

$= \int_0^{2\pi}\int_0^1 2\,r\,dr\,d\theta = [r^2]_0^1 \cdot 2\pi$

$= 2\pi.$

Solution

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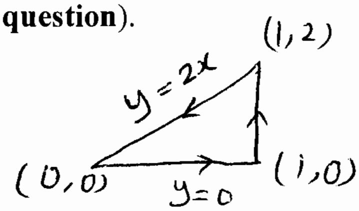
QUIZ # 2(b)

Name ____________________ ID # __________ Section # ____

Q1) In the following question, use *Green's theorem* to write R.H.S of the given line integral as a double integral, showing correct integral limits (**Do not evaluate integrals in this question**).

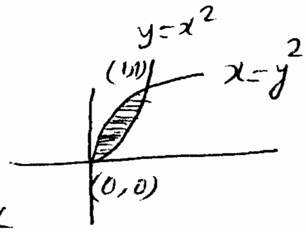
(a) $\oint_C -x^4y^2dx + x^2y^4dy = \int_0^1\int_0^{2x} 2x\,y^4 + 2x^4y \;dy\,dx$



where C is triangle counterclockwise with vertices A(0,0), B(1,0) and C(1,2).

R : $y=0$ to $y=2x$ and $x=0$ to $x=1$.

(b) $\oint_C (2y + x^2)dx - (3x - 4y^2)dy = \int_0^1\int_{\sqrt{y}}^{y^2} -5\,dx\,dy$



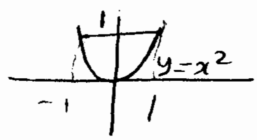
Where C is closed counter clockwise by $y = x^2, and\ x = y^2$.

R : $x=\sqrt{y}$ to $x=y^2$, $y=0$ to $y=1$ 3

OR You can use alternate description as: $\int_0^1\int_{x^2}^{\sqrt{x}} -5\,dy\,dx$

Q2) Evaluate the integral using **Green's theorem** $\oint_C 4y\,dx + 5x\,dy$

Where C is formed by $y = x^2, and\ y = 1$..



$$P = 4y, \quad \frac{\partial P}{\partial y} = 4 \qquad\qquad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 5-4$$
$$Q = 5x \quad \frac{\partial Q}{\partial x} = 5 \qquad\qquad = 1$$

I
$$\iint_R dA = \int_{-1}^{1}\int_{x^2}^{1} dy\,dx = \int_{-1}^{1}[y]_{x^2}^{1}\,dx$$
$$= \int_{-1}^{1}(1-x^2)\,dx$$
$$= \left[x - \frac{x^3}{3}\right]_{-1}^{1}$$
$$= \left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right) \qquad 4$$
$$= \frac{4}{3}.$$

Solution

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QUIZ # 2(c)

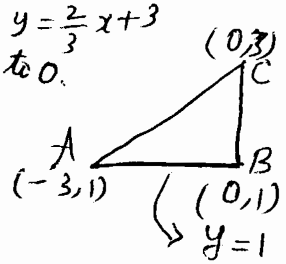
Name ______________________ ID # __________ Section # ____

Q1) In the following question, use *Green's theorem* to write R.H.S of the given line integral as a double integral, showing correct integral limits (**Do not evaluate integrals in this question**). 3

(a) $\oint_C x^3 y\,dx + xy^3\,dy = \int_{-3}^{0}\int_{1}^{\frac{2}{3}x+3} (y^3 - x^3)\,dy\,dx$

R: $y = 1$ to $y = \frac{2}{3}x + 3$

$x = -3$ to 0.

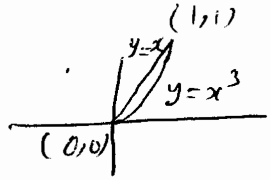


where C is triangle counterclockwise with vertices A(-3,1), B(0,1) and C(0,3).

Slope of AB $m = \frac{2}{3}$, Eqn of AB: $y = \frac{2}{3}x + 3$

(b) $\oint_C y\sin^2 x\,dx - x\cos^2 y\,dy = \int_0^1\int_{x^3}^{x} -(\cos^2 y + \sin^2 x)\,dy\,dx$

Where C is closed counter clockwise by $y = x^3, and\ x = y$



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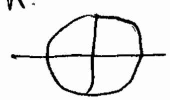
R: $y = x^3$ to $y = x$ and $x = 0$ to 1

You can also use $x = y$ and $x = y^{1/3}$, $y = 0$ to 1.

Q2) Evaluate the integral using **Green's theorem** $\oint_C x^2\,dy$

Where C is given by $x^2 + y^2 = 9$.

R: $x^2 + y^2 = 9$



$P = 0$, $Q = x^2$, $\frac{\partial Q}{\partial x} = 2x$

$$I = \iint_R 2x\,dA = \int_0^{2\pi}\int_0^3 2r\cos\theta\ r\,dr\,d\theta$$

$$= \int_0^{2\pi} \left[\frac{2}{3}r^3\right]_0^3 \cos\theta\,d\theta$$

$$= 18\left[\sin\theta\right]_0^{2\pi}$$

$$= 0.$$

$x = r\cos\theta$

$y = r\sin\theta$

$r = 0$ to 3

$\theta = 0$ to 2π

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