DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics Term o41

QUIZ # 1

Name ______________________ ID # __________ Sec # _____

In the following $\underline{r}$ represent the position vector, $\underline{a} = \langle a_1, a_2, a_3 \rangle$ is a constant vector, $\underline{F} = \langle F_1, F_2, F_3 \rangle$ is a vector field and *f(x,y,z)* is a scalar function. **Show** the following results.

Q1) $(\underline{a} \times \underline{\nabla}) \times \underline{r} = -2\ \underline{a}$.

First evaluate $(\underline{a} \times \underline{\nabla})$

$$(\underline{a} \times \underline{\nabla}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \underline{i}(a_2 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial y}) - \underline{j}(a_1 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial x}) + \underline{k}(a_1 \frac{\partial}{\partial y} - a_2 \frac{\partial}{\partial x})$$

Hence, $(\underline{a} \times \underline{\nabla}) \times \underline{\mathbf{r}} =$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_2 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial y} & -(a_1 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial x}) & a_1 \frac{\partial}{\partial y} - a_2 \frac{\partial}{\partial x} \\ x & y & z \end{vmatrix} = \underline{i}\left\{-(a_1 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial x})z - (a_1 \frac{\partial}{\partial y} - a_2 \frac{\partial}{\partial x})y\right\} -$$

$$\underline{j}\left\{(a_2 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial y})z - (a_1 \frac{\partial}{\partial y} - a_2 \frac{\partial}{\partial x})x\right\} + \underline{k}\left\{(a_2 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial y})y + (a_1 \frac{\partial}{\partial z} - a_3 \frac{\partial}{\partial x})x\right\}$$

$$= -2(a_1 \underline{i} + a_2 \underline{j} + a_3 \underline{k}) = -2\underline{a}.$$

Q2) curl(grad)*f(x,y,z)* = 0 → $\underline{\nabla} \times (\underline{\nabla} f) = 0$

$\underline{\nabla}.f = \frac{\partial f}{\partial x}\underline{i} + \frac{\partial f}{\partial y}\underline{j} + \frac{\partial f}{\partial z}\underline{k}$.**Hence**

$$\underline{\nabla} \times (\underline{\nabla} f) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix} = \underline{i}(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}) - \underline{j}(\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z})$$

$$+ \underline{k}(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}) = 0$$