DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics Term o41

QUIZ # 4 A

Name ______________________ ID # __________ Sec # _____

Q1). From the form of the functions given below, tell if the Fourier series be a sine or a cosine series? DO NOT FIND THE FOURIER SERIES.

(a) $f(x)=\begin{cases}1, & -1<x<0\\ -1, & 0<x<1.\end{cases}$ $f(x)$ is Odd. Fourier Sine series.

(b) $f(x)=x|x|,\quad -3<x<3$ $f(x) = (\text{odd})(\text{Even}) = \text{odd}$. Fourier Sine series

(c) $f(x)=x^2+1,\quad -1<x<1$ $f(x)$ is even. Fourier cosine Series

Q2) Expand the following in an appropriate cosine or sine or Fourier series.

$$f(x)=\begin{cases}\pi, & -\pi<x<-\pi/2\\ 0, & -\pi/2<x<\pi/2\\ \pi, & \pi/2<x<\pi\end{cases}$$

We have $f(-x) = f(x)$

Thus $b_n = 0$. ($f(x)$ even).

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\cancel{x}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \pi\,dx$$

(as $f(x)=0$ in $0<x<\frac{\pi}{2}$)

$$a_0 = \frac{2}{\cancel{\pi}}\cdot\cancel{\pi}\,\Big[x\Big]_{\pi/2}^{\pi} = 2\left[\pi - \frac{\pi}{2}\right] = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_{\pi/2}^{\pi} \pi\cos nx\,dx$$

(as $f(x)=0$ in $0<x<\pi/2$)

$$= 2\cdot\frac{1}{n}\Big[\sin nx\Big]_{\pi/2}^{\pi} = -\frac{2}{n}\sin n\frac{\pi}{2}$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{-2}{n}\sin\frac{n\pi}{2}\cos nx.$$

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QUIZ # 4 B

Name ____________________ ID # __________ Sec # _____

Q1). From the form of the functions given below, tell if the Fourier series be a sine or a cosine series? DO NOT FIND THE FOURIER SERIES.

(a) $f(x)=\begin{cases}-x, -1<x<0\\ x, 0<x<1.\end{cases}$ $f(x)$ is odd. Fourier Sine series.

(b) $f(x)=x\sin x,\quad -\pi<x<\pi$ $f(x) = (\text{odd})(\text{odd}) = \text{even}$. Fourier Cosine series.

(c) $f(x)=x^3+x,\quad -1<x<1$ $f(x)$ is odd. Fourier Sine series.

Q2) Expand the following in an appropriate cosine or sine or Fourier series.

$$f(x)=\begin{cases}1, & -4<x<-2\\ 0, & -2<x<2\\ 1, & 2<x<4\end{cases}$$

As $f(-x)=f(x)$, we expect a Fourier cosine series ($b_n=0$).

$p=4$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{4}$$

$$a_0 = \frac{1}{4}\int_{-4}^{4} f(x)\,dx = \frac{2}{4}\int_0^4 f(x)\,dx = \frac{1}{2}\int_2^4 1\,dx$$

($\because f(x)=0$ in $0<x<2$).

$$= \frac{1}{2}[x]_2^4 = \frac{1}{2}(4-2) = 1.$$

$$a_n = \frac{1}{4}\int_{-4}^{4} f(x)\cos\frac{n\pi}{4}x\,dx = \frac{2}{4}\int_0^4 f(x)\cos\frac{n\pi}{4}x\,dx$$

$$= \frac{1}{2}\int_2^4 \cos\frac{n\pi}{4}x\,dx. \quad (\text{as } f(x)=0 \text{ in } 0<x<2)$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{4}^{\,2}}{n}\left[\sin\frac{n\pi}{4}x\right]_2^4 = \frac{-2}{n}\sin\frac{n\pi}{2}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-2}{n}\sin\frac{n\pi}{2}\cos\frac{n\pi}{4}x.$$

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QUIZ # 4 C

Name ____________________ ID # __________ Sec # _____

Q1). From the form of the functions given below, tell if the Fourier series be a sine or a cosine series? DO NOT FIND THE FOURIER SERIES.

(a) $f(x)=\begin{cases}-x^2-1, -1<x<0\\ x^2+1, 0<x<1.\end{cases}$ $f(x)$ is odd. Fourier sine series.

(b) $f(x)=x\cos 3x,\quad -1<x<1$ $f(x) = (\text{odd})(\text{even}) = \text{odd}$. Fourier sine series.

(c) $f(x)=x^4+4,\quad -2<x<2$ $f(x)$ even. Fourier cosine series.

Q2) Expand the following in an appropriate cosine or sine or Fourier series.

$$f(x)=\begin{cases}-2, & -2<x<-1\\ 0, & -1<x<1\\ 2, & 1<x<2\end{cases}$$

$f(x)$ is odd function. We expect a Fourier sine series i.e. $a_0=a_n=0$.

$p=2$.

$$f(x)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{2}x.$$

$$b_n=\frac{1}{2}\int_{-2}^{2}\underbrace{f(x)}_{\text{odd}}\underbrace{\sin\frac{n\pi}{2}x}_{\text{odd}}\,dx = \frac{1}{\cancel{2}}\cdot\cancel{2}\int_0^2 f(x)\sin\frac{n\pi}{2}x\,dx$$

$$=\int_1^2 2\cdot\sin\frac{n\pi}{2}x\,dx \quad (f(x)=0 \text{ in } 0<x<1).$$

$$=2\cdot\left(\frac{-2}{n\pi}\right)\left[\cos\frac{n\pi}{2}x\right]_1^2 = -\frac{4}{n\pi}\cos n\pi = \frac{(-1)^{n+1}4}{n\pi}$$

$$f(x)=\sum_{n=1}^{\infty}\frac{-4}{n\pi}\cos n\pi\,\sin\frac{n\pi}{2}x = \sum_{n=1}^{\infty}(-1)^{n+1}4\sin\frac{n\pi}{2}x.$$