DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics Term o41

QUIZ # 3 (version 1)

Name _____ ID # _____ Sec # _____

Q1) Find Laplace transform of the following functions. Remember: $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

(a) $f(t) = t(e^t + e^{-t})^2$ $= t(e^{2t} + e^{-2t} + 2) = te^{2t} + te^{-2t} + 2t.$

As $\mathcal{L}\{t\} = \frac{1}{s^2}$, we get using above result

$$\mathcal{L}\{f(t)\} = \frac{1}{(s-2)^2} + \frac{1}{(s+2)^2} + \frac{2}{s^2}.$$

(b) $f(t) = \begin{cases} t, 0 \le t < 1 \\ 1, t \ge 1 \end{cases}$ Remember: $\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$

$f(t) = t - (t-1)\,u(t-1)$

$f(t) = t$

$F(s) = \frac{1}{s}$

Using above property,

$$\mathcal{L}\{f(t)\} = \frac{1}{s} - \frac{e^{-s}}{s-1}.$$

Q2) Find inverse Laplace transform

$$F(s) = \frac{s}{s^2 + 4s + 5} = \frac{s}{s^2+4s+4+1} = \frac{s}{(s+2)^2+1}$$

$$= \frac{s+2-2}{(s+2)^2+1} = \frac{s+2}{(s+2)^2+1} - \frac{2}{(s+2)^2+1}$$

$$\mathcal{L}^{-1}\{F(s)\} = e^{-2t}\cos t - 2e^{-2t}\sin t \qquad s \to s+2$$

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QUIZ # 3 (version 2)

Name _____ ID # _____ Sec # _____

Q1) Find Laplace transform of the following functions.

Remember: $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

(a) $f(t) = e^{-2t}(\sin t + 2\cos t)$ $= e^{-2t}\sin t + 2e^{-2t}\cos t$

As $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$, $\mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$, We use above result to write

$$\mathcal{L}\{f(t)\} = \frac{1}{(s+2)^2+1} + 2\frac{(s+2)}{(s+2)^2+1}$$

(b) $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ e^{2t}, & t \ge 2 \end{cases}$: Remember: $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$.

$f(t) = e^{2t}u(t-2) = e^4 e^{2(t-2)}u(t-2)$.

$\rightarrow f(t) = e^{2t} \Rightarrow F(s) = \frac{1}{s-a}$

Using above result,

$$\mathcal{L}\{f(t)\} = e^4\frac{e^{-2s}}{(s-2)}$$

Q2) Find inverse Laplace transform

$$F(s) = \frac{s}{s^2+4s+5} = \frac{s}{s^2+4s+4+1} = \frac{s}{(s+2)^2+1} = \frac{s+2-2}{(s+2)^2+1}$$

$$= \frac{s+2}{(s+2)^2+1} - \frac{2}{(s+2)^2+1}$$

Shifting $s \to s+2$

$$\mathcal{L}^{-1}\{F(s)\} = e^{-2t}\cos t - 2e^{-2t}\sin t.$$

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QUIZ # 3 (version 3)

Name _____ ID # _____ Sec # _____

Q1) Find Laplace transform of the following functions.

Remember: $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

(a) $f(t) = t^2(1-e^{2t})^2 = t^2(1 + e^{4t} - 2e^{2t})$

As $\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\} = \frac{2}{s^3}$, We have by above result

$\mathcal{L}\{t^2 + t^2e^{4t} - 2t^2e^{2t}\} = \frac{2}{s^3} + \frac{2}{(s-4)^3} - \frac{4}{(s-2)^3}$.

Remember: $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$.

(b) $f(t) = \begin{cases} \sin t, & 0 \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$

$f(t) = \sin t - \sin t\, u(t-2\pi) = \sin t - \sin(t-2\pi)u(t-2\pi)$

As $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$, we have, using above result

$\mathcal{L}\{f(t)\} = \frac{1}{s^2+1} + \frac{e^{-2\pi s}}{s^2+1}$.

→ $f(t) = \sin t$, $F(s) = \frac{1}{s^2+1}$.

Q2) Find inverse Laplace transform

$F(s) = \frac{2s}{s^2+6s+10} = \frac{2s}{(s^2+6s+9)+1} = \frac{2s}{(s+3)^2+1} = \frac{2(s+3)-6}{(s+3)^2+1}$

$= 2\frac{(s+3)}{(s+3)^2+1} - \frac{6}{(s+3)^2+1}$

By shifting (translation) on s-axis $s \to s+3$

$\mathcal{L}^{-1}\{F(s)\} = 2e^{-3t}\cos t - 6e^{-3t}\sin t$.