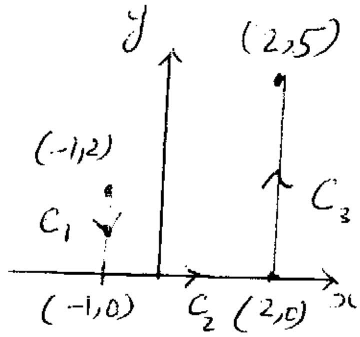
(9.8)

10) $I = \int_C (2x+y)\,dx + xy\,dy.$




$C = C_1 \cup C_2 \cup C_3$

On $C_1$ : We have $x=-1$, $0 \le y \le 2$ ; $dx = 0$

$$I = -\int_0^2 (-1)\,y\,dy = -\left[-\frac{y^2}{2}\right]_0^2 = +2 \quad — (1)$$

On $C_2$ $y=0$, $x=-1$ to $x=2$ ; $dy=0$

$$I = \int_{-1}^{2} (2x)\,dx = \left[x^2\right]_{-1}^{2} = 4-1 = 3 \quad — (2)$$

On $C_3$ $0 \le y \le 5$; $x=2$, $dx=0$

$$I = \int_0^5 2y\,dy = \left[y^2\right]_0^5 = 25 \quad — (3)$$

Given integral $= +2+3+25 = 30.$

16) $\int_C -y^2\,dx + xy\,dy$

C: $x = 2t$, $y = t^3$, $0 \le t \le 2$

$\frac{dx}{dt} = 2$, $\frac{dy}{dt} = 3t^2$

$$\int_C -y^2\,dx + xy\,dy = \int_0^2 -t^6 \cdot 2\,dt + 2t^4(3t^2) \times dt$$

$$= \left[-\frac{2}{7}t^7 + \frac{6}{7}t^7\right]_0^2$$

$$= \frac{4}{7}\cdot 2^7 = \frac{1}{7}\,2^9.$$

30) $F = e^x i + x e^{xy} j + xy\,e^{xyz} k$

$r = t i + t^2 j + t^3 k$, $0 \le t \le 1$

$$\int_C F\cdot r = \int_0^1 F(t)\cdot \frac{dr}{dt}\,dt$$

$$F(t) = e^t i + t e^{t^3} j + t^3 e^{t^6} k$$

$$\frac{dr}{dt} = i + 2t j + 3t^2 k$$

$$F(t)\cdot \frac{dr}{dt} = e^t + 2t^2 e^{t^3} + 3t^5 e^{t^6}$$

Thus

$$\int_C F\cdot r = \int_0^1 \left(e^t + 2t^2 e^{t^3} + 3t^5 e^{t^6}\right)dt$$

$$= \left[e^t + \frac{2}{3}e^{t^3} + \frac{1}{2}e^{t^6}\right]_0^1$$

$$= e + \frac{2}{3}e + \frac{1}{2}e - \left(1 + \frac{2}{3} + \frac{1}{2}\right)$$

$$= \frac{13}{6}(e-1).$$