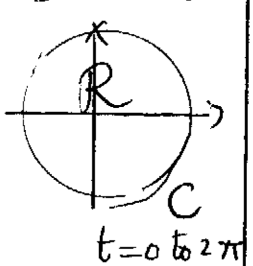
3) $\oint_C -y^2\,dx + x^2\,dy$

$$= \iint_R (2x+2y)\,dA$$

$C:\ x = 3\cos t,\ y = 3\sin t$



L.H.S.

$\frac{dx}{dt} = -3\sin t$

$\frac{dy}{dt} = 3\cos t$

We get

$$\int_0^{2\pi} \{-9(\sin^2 t)(-3\sin t) + 9(\cos^2 t)\,3\cos t\}\,dt$$

$$= 27\int_0^{2\pi} \{(1-\cos^2 t)(\sin t) + (1-\sin^2 t)\cos t\}\,dt$$

$$= 27\left(-\cos t + \frac{1}{3}\cos^3 t + \sin t - \frac{1}{3}\sin^3 t\right)_0^{2\pi}$$

$$= 0.$$

R.H.S. Use polar coordinates for region inside circle

$x = r\cos\theta$, $y = r\sin\theta$; $r = 0$ to 3, $\theta = 0$ to 2π

We get

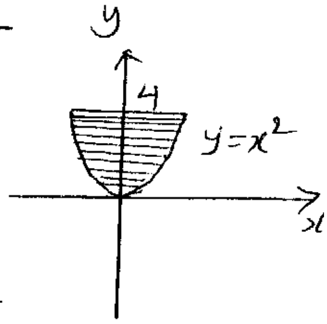
$$2\int_0^{2\pi}\int_0^3 (\cos\theta + \sin\theta)\, r\,dr\,d\theta$$

$$= 2\cdot\left[\frac{r^2}{2}\right]_0^3 \{\sin\theta - \cos\theta\}_0^{2\pi}$$

$$= 9\cdot 0 \qquad = 0.$$

6)

$\oint_C (x^2+y^2)\,dx + (2x^2-y)\,dy$



$P = x^2+y^2,\ Q = 2x^2-y$

$C:\ y = x^2,\ y = 4$

By Green's theorem the integral is

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial x}\right) dA$$

over region: $y = x^2$ to $y = 4$
$x = -2$ to $x = 2$.

This gives

$$\int_{-2}^{2}\int_{y=x^2}^{4} (4x - 2y)\,dy\,dx = -\frac{96}{5}$$

18) $\frac{1}{2}\oint_C -y\,dx + x\,dy = $ Area of region R.

Here $P = -\frac{1}{2}y,\ Q = \frac{1}{2}x$

$\frac{\partial P}{\partial y} = -\frac{1}{2},\quad \frac{\partial Q}{\partial x} = \frac{1}{2}$

By Green's theorem, the integral is

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_R \left(\frac{1}{2} + \frac{1}{2}\right) dA$$

$$= \iint_R dA$$

$$= \text{Area of } R.$$

~~25) $\oint \dfrac{\ldots\, y^2\,dy}{(x^2+y^2)^2}$, $C: x^2+4y^2=4$~~

~~$P = -\dfrac{y^3}{(x^2+y^2)^2}$, $Q = \dfrac{xy^2}{(x^2+y^2)^2}$; $\dfrac{\partial P}{\partial y} \neq \dfrac{\partial Q}{\partial x}$~~

~~Idea in Example 6 can not be applied.~~