(1.4)

10) $(1+x^2)\frac{dy}{dx} = (1+y)^2$

Separating variables by cross multiplying

$$\frac{dy}{(1+y)^2} = \frac{dx}{1+x^2}$$

Integrating both sides,

$$\int \frac{dy}{(1+y)^2} = \int \frac{dx}{1+x^2}$$

$$\Rightarrow \frac{-1}{(1+y)} = \tan^{-1}x + C$$

$$\text{or} \quad 1+y = \frac{-1}{\tan^{-1}x + C}$$

$$\Rightarrow y = -1 \; \frac{-1}{\tan^{-1}x + C}.$$

24) $\tan x \frac{dy}{dx} = y$; $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

$$\text{D.E.} \Rightarrow \frac{dy}{y} = \frac{dx}{\tan x}$$

$$\text{or} \quad \frac{dy}{y} = \cot x \; dx$$

Integrating both sides,

$$\ln y = \ln(\sin x) + C_1$$

$$\text{or} \quad y = e^{\ln(\sin x) + C_1} = e^{C_1} e^{\ln(\sin x)}$$

$$\text{or} \quad y = C \sin x \quad (C = e^{C_1})$$

$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2}$ and $y = \frac{\pi}{2}$

i.e. $\frac{\pi}{2} = C \sin\frac{\pi}{2} \Rightarrow C = \frac{\pi}{2}$

Hence, $y = \frac{\pi}{2}\sin x$

27) $\frac{dy}{dx} = 6e^{2x-y}$, $y(0) = 0$

$$\text{or} \quad \frac{dy}{dx} = 6e^{2x}e^{-y}$$

$$\text{or} \quad e^{y}\,dy = 6e^{2x}\,dx$$

Integrating both sides,

$$e^{y} = 3e^{2x} + C$$

Using $y(0) = 0$

$$1 = 3 + C \Rightarrow C = -2$$

Hence, $e^{y} = 3e^{2x} - 2$.

29) Let $P(t)$ be population

$P(0) = 25{,}000$ in 1960

$P(10) = 30{,}000$ in 1970

$P(40) = ?$ in 2000

$$\frac{dP}{dt} = KP \Rightarrow$$

$$P = Ce^{Kt}$$

$$P(0) = 25{,}000 \Rightarrow C = 25{,}000$$

Thus $P(t) = 25{,}000\,e^{Kt}$

$$P(10) = 30{,}000 \Rightarrow$$

$$30000 = 25000\,e^{10K}$$

$$\text{or} \quad e^{10K} = \frac{30000}{25000} = \frac{6}{5}$$

$$10K = \ln\left(\frac{6}{5}\right) \Rightarrow K = \frac{1}{10}\ln\left(\frac{6}{5}\right)$$

$$\Rightarrow P(t) = 25{,}000\,e^{t\ln\left(\frac{6}{5}\right)^{1/10}} = \ln\left(\frac{6}{5}\right)^{1/1}$$

$$= 25{,}000\left(\frac{6}{5}\right)^{t/10}$$

$P(40)$ can be found $(t = 40)$.

Math 260
(1.5)

4) $y' - 2xy = e^{x^2}$

Linear D.E.

$P(x) = -2x$.

I) I.F. $= e^{\int -2x\,dx} = e^{-x^2}$

Multiply D.E. by e^{-x^2}

$$e^{-x^2} y' - 2x e^{-x^2} y = 1.$$

II) $\Rightarrow \frac{d}{dx}\left(e^{-x^2} y\right) = 1$

Integrating,

$$e^{-x^2} y = x + C$$

or $y = x e^{x^2} + C e^{x^2}$.

12) $xy' + 3y = 2x^5$, $y(2) = 1$.

Linear D.E.

Re-write in standard form

$$y' + \frac{3}{x} y = 2x^4.$$

$P(x) = \frac{3}{x}$

I.F. $= e^{\int \frac{3}{x} dx} = e^{3\ln x} = x^3$

Multiply D.E. by x^3

$$x^3 y + 3x^2 y = 2x^7$$

or $\frac{d}{dx}(y x^3) = 2x^7$

Integration gives

$$y x^3 = \frac{1}{4} x^8 + C$$

$y(2) = 1 \Rightarrow 8 = 64 + C \Rightarrow C = -56$

so $y x^3 = \frac{1}{4} x^8 - 56$.

28)

$(1 + 2xy) \frac{dy}{dx} = 1 + y^2$

Not linear in y. We re-write it in terms of $\frac{dx}{dy}$ as

$$(1+y^2)\frac{dx}{dy} - 2xy = 1$$

or $\frac{dx}{dy} - \frac{2y}{1+y^2} x = \frac{1}{1+y^2}$ (*)

This is linear in x.

I.F. $= e^{\int -\frac{2y}{1+y^2} dy} = e^{-\ln(1+y^2)}$

$= e^{\ln(1+y^2)^{-1}} = \frac{1}{1+y^2}$

Multiply (*) by $\frac{1}{1+y^2}$

$$\frac{1}{1+y^2}\frac{dx}{dy} - \frac{2y}{(1+y^2)^2} x = \frac{1}{(1+y^2)^2}$$

or $\frac{d}{dy}\left(\frac{1}{1+y^2} x\right) = \frac{1}{(1+y^2)^2}$

Integrating

$$\frac{1}{1+y^2} x = \int \frac{1}{(1+y^2)^2} dy$$

On R.H.S Put $y = \tan u$

$dy = \sec^2 u\, du$

$(1+y^2)^2 = (1+\tan^2 u)^2 = \sec^4 u$

R.H.S $= \int \frac{du}{\sec^2 u} = \int \cos^2 u\, du$

$= \int \frac{1 + \cos 2u}{2} du$

$= \frac{u}{2} + \frac{\sin 2u}{4}$

$= \frac{\tan^{-1} y}{2} + \frac{\sin 2(\tan^{-1} y)}{4}$

etc.