

Assignment # 5 MATH 513

- 1) Consider $u(x) = J_0(\alpha x)$ and $v(x) = J_0(\beta x)$.
Write the differential equations satisfied by $u(x)$ and $v(x)$
 $xu'' + u' + \alpha^2 x u = 0,$
 $xv'' + v' + \beta^2 x v = 0.$

Multiply the first by v and second by u and subtract to show that

$$(\beta^2 - \alpha^2) \int x J_0(\alpha x) J_0(\beta x) dx = x[\alpha J_0(\alpha x) J_0(\beta x) - \beta J_0(\alpha x)' J_0(\beta x)']$$

- 2) Use recurrence relation to write $J_3(x)$ in terms of J_1 and J_0 .
3) Evaluate $\int x^3 J_0(x) dx$ and $\int J_5(x) dx$
4) Show that

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$$

$$J_{-3/2}(x) = -\sqrt{\frac{2}{\pi x}} \left(\frac{\cos x}{x} + \sin x \right)$$

- 4) Using the general form write solution of the DE

$$y'' - \frac{5}{x} y' + \left(64x^6 + \frac{5}{x^2} \right) y = 0$$