MATH 301 Methods of Applied Mathematics

Section 9.1

We assume that we have parametric form of a smooth curve C given by

$$\underline{r}(t) = x(t)\underline{i} + y(t)\underline{j} + z(t)\underline{k}$$

$$= \langle x(t), y(t), z(t) \rangle$$

$$a \le t \le b.$$

If the equation is given in a non-parametric form then we can put it in a suitable parametric form.

Differentiation: $\underline{r}'(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$

Del Operator: $\underline{\nabla} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

We use this operator to define three operations

Gradient: Gradient of a scalar function *f(x,y,z)* is defined as

$$grad\, f = \underline{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle.$$

Divergence: Divergence of a vector field is a scalar function defined as

$$div\underline{F} = \underline{\nabla}.\underline{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}.$$

Curl: The curl of a vector field is given by

$$curl\underline{F} = \underline{\nabla} \times \underline{F} = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{bmatrix}$$

A vector field $\underline{F}$ is called *IRROTATIONAL* if $curl\underline{F} = 0$.