

(9.1)

12) $x=t, y=2x, x^2+y^2-z^2=1$

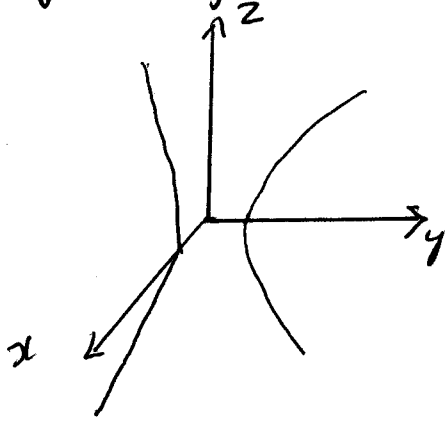
This gives

$x=t$

$y=2t$

$z = \sqrt{x^2+y^2-1} = \sqrt{5t^2-1}$

$$\underline{r}(t) = t \underline{i} + 2t \underline{j} + \sqrt{5t^2-1} \underline{k}$$



41) Length of the curve

$$\underline{r}(t) = a \cos t \underline{i} + a \sin t \underline{j} + ct \underline{k}$$

$0 \leq t \leq 2\pi$

$$s = \int_0^{2\pi} \|\underline{r}'(t)\| dt$$

$$\underline{r}'(t) = -a \sin t \underline{i} + a \cos t \underline{j} + c \underline{k}$$

$$\|\underline{r}'(t)\| = \sqrt{a^2 \sin^2 t + a^2 \cos^2 t + c^2} = \sqrt{a^2 + c^2}$$

we find

$$s = \int_0^{2\pi} (a^2 + c^2)^{1/2} dt$$

$$= (a^2 + c^2)^{1/2} \int_0^{2\pi} dt$$

$$= (a^2 + c^2)^{1/2} [t]_0^{2\pi} = 2\pi (a^2 + c^2)^{1/2}$$

26) Find equation of tangent vector

$x = t^3 - t, y = \frac{6t}{t+1}, z = (2t+1)^2; t=1$

At $t=0, P(x_0, y_0, z_0) = P(0, 3, 9)$

The vector along tangent line is given by

$$\underline{r}'(t) = (3t^2-1)\underline{i} + \frac{6}{(t+1)^2}\underline{j} + (8t+4)\underline{k}$$

$$= 2\underline{i} + \frac{3}{2}\underline{j} + 12\underline{k}$$

The eqn is

$x = x_0 + a_1 t = 0 + 2t$

$y = y_0 + a_2 t = 3 + \frac{3}{2}t$

$z = z_0 + a_3 t = 9 + 12t$

(28) $\frac{d}{dt} [\underline{r}(t) \cdot (t \underline{r}(t))]$

$$= \underline{r}(t) \cdot \frac{d}{dt} (t \underline{r}(t)) + \frac{d\underline{r}}{dt} \cdot (t \underline{r}(t))$$

$$= \underline{r}(t) \cdot \underline{r}(t) + t \underline{r}(t) \cdot \underline{r}'(t) + t \underline{r}'(t) \cdot \underline{r}(t)$$

$$= \underline{r}(t) \cdot \underline{r}(t) + 2t \underline{r}(t) \cdot \underline{r}'(t)$$

$$[\text{As } \underline{r}(t) \cdot \underline{r}'(t) = \underline{r}'(t) \cdot \underline{r}(t)]$$

4) $\underline{r}(t) = 4\underline{i} + 2 \cos t \underline{j} + 3 \sin t \underline{k}$

Hence

$x(t) = 4$

$$\left. \begin{aligned} y(t) &= 2 \cos t \\ z(t) &= 3 \sin t \end{aligned} \right\} \frac{y^2}{4} + \frac{z^2}{9} = 1$$

This is an ellipse in yz -plane at $x=4$

