12) $x = t,\quad y = 2x,\quad x^2+y^2-z^2=1$

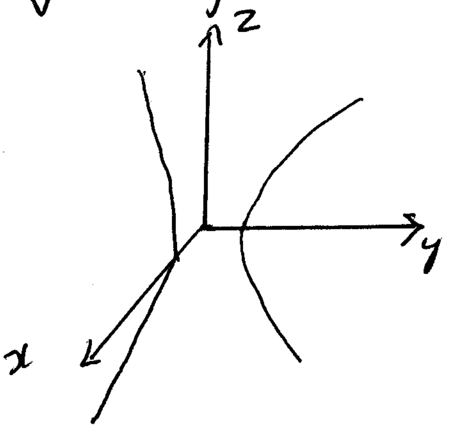
This gives

$x = t$

$y = 2t$

$z = \sqrt{x^2+y^2-1} = \sqrt{5t^2-1}$

$\left.\right\} \underline{r}(t) = t\,\underline{i} + 2t\,\underline{j} + \sqrt{5t^2-1}\;\underline{k}$



26) Find equation of tangent vector

$x = t^3 - t,\quad y = \dfrac{6t}{t+1},\quad z = (2t+1)^2;\ t=1$

At $t=0$, $P(x_0, y_0, z_0) = P(0, 3, 9)$

The vector along tangent line is given by

$\underline{r}'(t)\Big|_{t=[redacted]} = (3t^2-1)\underline{i} + \dfrac{6}{(t+1)^2}\underline{j} + (8t+4)\underline{k}\Big|_{t=[redacted]}$

$= 2\underline{i} + \dfrac{3}{2}\underline{j} + 12\,\underline{k}$

The eqn is

$x = x_0 + a_1 t = 0 + 2t$

$y = y_0 + a_2 t = 3 + \frac{3}{2}t$

$z = z_0 + a_3 t = 9 + 12t$

(28) $\dfrac{d}{dt}\left[\underline{r}(t)\cdot(t\,\underline{r}(t))\right]$

$= \underline{r}(t)\cdot\dfrac{d}{dt}(t\,\underline{r}(t)) + \dfrac{d\underline{r}}{dt}\cdot(t\,\underline{r}(t))$

$= \underline{r}(t)\cdot\underline{r}(t) + t\,\underline{r}(t)\cdot\underline{r}'(t) + t\,\underline{r}'(t)\cdot\underline{r}(t)$

$= \underline{r}(t)\cdot\underline{r}(t) + 2t\,\underline{r}(t)\cdot\underline{r}'(t)$

$[\text{As } \underline{r}(t)\cdot\underline{r}'(t) = \underline{r}'(t)\cdot\underline{r}(t)]$

41) Length of the curve

$\underline{r}(t) = a\cos t\,\underline{i} + a\sin t\,\underline{j} + c\,t\,\underline{k}$

$0 \le t \le 2\pi.$

$$s = \int_0^{2\pi} \|\underline{r}'(t)\|\,dt$$

As $\underline{r}'(t) = -a\sin t\,\underline{i} + a\cos t\,\underline{j} + c\,\underline{k}$

$\|\underline{r}'(t)\| = \sqrt{a^2\sin^2 t + a^2\cos^2 t + c^2}$

$= \sqrt{a^2+c^2}$

we find

$$s = \int_0^{2\pi} (a^2+c^2)^{1/2}\,dt$$

$= (a^2+c^2)^{1/2}\int_0^{2\pi} dt$

$= (a^2+c^2)^{1/2}\,[t]_0^{2\pi}$

$= 2\pi\,(a^2+c^2)^{1/2}.$

4) $\underline{r}(t) = 4\underline{i} + 2\cos t\,\underline{j} + 3\sin t\,\underline{k}$

Hence

$x(t) = 4$

$y(t) = 2\cos t$

$z(t) = 3\sin t$

$\left.\right\}\ \dfrac{y^2}{4} + \dfrac{z^2}{9} = 1$

This is an ellipse in yz-plane at $x = 4$