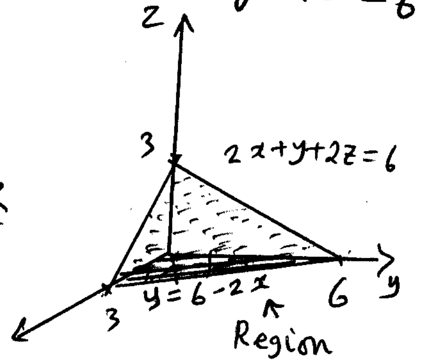
(9.14)

(3) $\underline{F} = z\underline{i} + x\underline{j} + y\underline{k}$, S: portion of plane $2x+y+2z=6$ in the first octant.



$$\text{Curl } \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix} = \underline{i} + \underline{j} + \underline{k}$$

$$\underline{n} = \frac{\langle 2, 1, 2\rangle}{\sqrt{4+1+4}} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$$

$$\text{Curl } \underline{F} \cdot \underline{n} = \frac{2}{3} + \frac{1}{3} + \frac{2}{3} = \frac{5}{3}.$$

The surface is : $z = 3 - \frac{1}{2}y - x$, so that

$$ds = \sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + 1 + \frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}.$$

The surface integral becomes

$$\int_0^3 \int_{6-2x}^{6} \frac{5}{\cancel{3}} \cdot \frac{\cancel{3}}{2} \, dy\, dx = \frac{5}{2} \int_0^3 (\cancel{6} - \cancel{6} + 2x)\, dx = \frac{45}{2}.$$

(17) Use Stokes theorem to evaluate $\oint z^2 e^{x^2} dx + xy^2 dy + \tan^{-1} y \, dz$

C: circle $x^2+y^2=9$, by finding surface S with C as boundary.

We can take S to be surface $z=0$, $x^2+y^2=9$.

We integrate surface integral $\iint_S (\text{curl } \underline{F}) \cdot n \, ds$ over this surface

Here $\text{curl } \underline{F} = \frac{1}{1+y^2}\underline{i} + (2ze^{x^2})\underline{j} + y^2 \underline{k}$

$\underline{n}$ = normal to xy plane = $\underline{k}$ so $(\text{curl } \underline{F}) \cdot \underline{n} = y^2$

$I = \iint_R y^2 \, dA$. For this region we use polar coordin

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \begin{aligned} r &= 0 \text{ to } 3 \\ \theta &= 0 \text{ to } 2\pi \end{aligned}$$

$$I = \int_0^{2\pi}\int_0^3 r^2 \sin^2\theta \, r\,dr\,d\theta$$

$$= \frac{1}{4}\left[r^4\right]_0^3 \int_0^{2\pi} \sin^2\theta \, d\theta$$

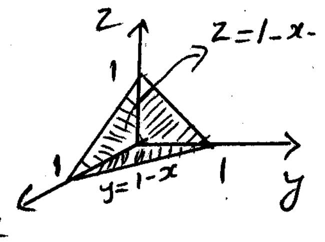
$$= \frac{81}{4}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \frac{81}{4}\pi.$$

For Q #11 see class work.

2) $\underline{F} = 6xy\,\underline{i} + 4yz\,\underline{j} + x\,e^{-y}\,\underline{k}$

D : region bounded by three coordinate planes and Plane $x+y+z=1$. By Divergence Theorem

$$\iint_S (\underline{F}\cdot\underline{n})\,ds = \iiint_V (\mathrm{div}\,\underline{F})\,dV$$



$\mathrm{div}\,\underline{F} = 6y + 4z.$

$$R.H.S = \iiint_V (6y+4z)\,dv = \int_0^1\int_0^{1-x}\int_0^{1-x-y}(6y+4z)\,dz\,dy\,dx$$

$$= \int_0^1\int_0^{1-x}\Big[(6yz+2z^2)\Big]_{z=0}^{1-x-y} dy\,dx \longrightarrow \frac{5}{12}.$$

④ $\underline{F} = 4x\,\underline{i} + y\,\underline{j} + 4z\,\underline{k}$, D: region bounded by sphere $x^2+y^2+z^2=4$.

$\mathrm{div}\,\underline{F} = 4+1+4 = 9.$

$$\iint_S (\underline{F}\cdot\underline{n})\,ds = \iiint_V 9\,dV$$

Using spherical coordinates, the volume is described as $\rho = 0$ to $\rho = 2$; $\phi = 0$ to π and $\theta = 0$ to 2π.

$$\therefore I = 9\int_0^{2\pi}\int_0^{\pi}\int_0^{2}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = 9\cdot\frac{1}{3}\int_0^{2\pi}\int_0^{\pi}\left[\rho^3\right]_0^2\sin\phi\,d\phi\,d\theta$$

$$= 9\cdot\frac{8}{3}\left[-\cos\phi\right]_0^{\pi}\int_0^{2\pi}d\theta = 9\cdot\frac{16}{3}\times 2\pi = 96\cdot\pi.$$