24) $\underline{a}$ is given to be a constant vector and

$$\underline{r} = x\,\underline{i} + y\,\underline{j} + z\,\underline{k}$$

$$\underline{a} = \langle a_1, a_2, a_3 \rangle .$$

To find

$$\nabla \cdot \left[(\underline{r} \cdot \underline{r})\,\underline{a} \right]$$

Now

$$\underline{r} \cdot \underline{r} = \langle x, y, z \rangle \cdot \langle x, y, z \rangle$$
$$= x^2 + y^2 + z^2$$

$$(\underline{r} \cdot \underline{r})\,\underline{a} = x^2+y^2+z^2 \; \langle a_1, a_2, a_3 \rangle$$
$$= \left\langle a_1(x^2+y^2+z^2),\ a_2(x^2+y^2+z^2),\ a_3(x^2+y^2+z^2) \right\rangle$$

So $\nabla \cdot \left[(\underline{r} \cdot \underline{r})\,\underline{a} \right] =$

$$\frac{\partial}{\partial x} a_1(x^2+y^2+z^2) + \frac{\partial}{\partial y} a_2(x^2+y^2+z^2) + \frac{\partial}{\partial z} a_3(x^2+y^2+z^2)$$

$$= 2(a_1 x + a_2 y + a_3 z)$$

$$= 2 \langle a_1, a_2, a_3 \rangle \cdot \langle x, y, z \rangle$$

$$= 2\ \underline{a} \cdot \underline{r}$$
$$= 2\ \underline{r} \cdot \underline{a}$$

9.7

28) $\nabla \times (f\,\underline{F})$

Here $\underline{F} = \langle F_1, F_2, F_3 \rangle$

$$f\,\underline{F} = \langle f F_1 + f F_2 , f F_3 \rangle$$

$$\nabla \times (f\,\underline{F}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ fF_1 & fF_2 & fF_3 \end{vmatrix}$$

$$= \underline{i} \left\{ \frac{\partial}{\partial y}(fF_3) - \frac{\partial}{\partial z}(fF_2) \right\} + \underline{j} \left\{ \frac{\partial}{\partial z}(fF_1) - \frac{\partial}{\partial x}(fF_3) \right\} + \underline{k} \left\{ \frac{\partial}{\partial x}(fF_2) - \frac{\partial}{\partial y}(fF_1) \right\}$$

Collecting term after differentiation by product rule

$$f\left[\left\{ \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right\} \underline{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x} \right) \underline{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \underline{k} \right]$$

$$+ \left[\left(\frac{\partial f}{\partial y} F_3 - \frac{\partial f}{\partial z} F_2 \right) \underline{i} + \left(\frac{\partial f}{\partial z} F_1 - \frac{\partial f}{\partial x} F_3 \right) \underline{j} + \left(\frac{\partial f}{\partial x} F_2 - \frac{\partial f}{\partial y} F_1 \right) \underline{k} \right]$$

$$= f(\nabla \times \underline{F}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= f(\nabla \times F) + (\nabla f) \times \underline{F} .$$