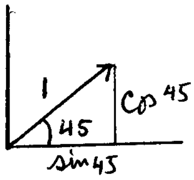
10) The unit vector making angle of 45° with x-axis has components $|u|\cos 45^\circ$, $|u|\sin 45^\circ$



$\hat{u} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$

$= \langle 3, -2y \rangle$

$D_{\underline{u}} f = \langle 3, -2y \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{3}{\sqrt{2}} - \sqrt{2}\, y.$

9.5

14) The unit vector along the given vector $6\underline{i} + 8\underline{j}$ is

$\underline{u} = \frac{6}{\sqrt{6^2+8^2}}\underline{i} + \frac{8}{\sqrt{6^2+8^2}}\underline{j}$

$= \frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$

$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$

$= \langle \frac{y(x+y) - xy}{(x+y)^2}, \frac{x(x+y) - xy}{(x+y)^2} \rangle$

$= \langle \frac{y^2}{(x+y)^2}, \frac{x^2}{(x+y)^2} \rangle$

$\nabla f|_{(2,-1)} = \langle 1, 4 \rangle$

$D_{\underline{u}} f = \nabla f|_{(2,-1)} \cdot \underline{u}$

$= \langle 1, 4 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle$

$= \frac{3}{5} + \frac{16}{5} = \frac{19}{5}.$

22) We need to find $D_{\underline{u}} f|_{P(1,1)}$ where $\underline{u}$ is unit vector along $\overrightarrow{PQ}$

$\underline{u} = \frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \frac{\langle -1, -1, 6-1 \rangle}{\sqrt{4+25}}$

$= \langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \rangle.$

$\nabla f = (3x^2 - 5y)\underline{i} - (5x - 2y)\underline{j}$

$\nabla f|_{P(1,1)} = \langle -2, -3 \rangle.$

Thus $D_{\underline{u}} f = \langle -2, -3 \rangle \cdot \langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \rangle$

$= \frac{4}{\sqrt{29}} - \frac{15}{\sqrt{29}} = \frac{-11}{\sqrt{29}}.$

30) The direction of most rapid decrease is $-\nabla f|_{P(\frac{1}{2}, \frac{1}{6}, \frac{1}{3})}$

Now

$-\nabla f|_P = -\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle|_P$

$= -\langle \frac{z}{xy}\frac{y}{z}, \frac{z}{xy}\frac{x}{z}, \frac{z}{xy}(-\frac{xy}{z^2}) \rangle|_P$

$= -\langle \frac{1}{x}, \frac{1}{y}, -\frac{1}{z} \rangle|_{P(\frac{1}{2}, \frac{1}{6}, \frac{1}{3})}$

$= -\langle 2, 6, -3 \rangle$

$= \langle -2, -6, 3 \rangle$

The minimum rate $-\|\nabla f\|$

$= -[4 + 36 + 9]^{1/2}$

$= -7$

(minus sign for rate of decrease)