

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 572 , Term: 081
Instructor: Dr. Faisal Fairag
Assignment (7)
Due Sunday 28/12/2008

1) Consider the following linear system

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 3 \end{bmatrix}$$

- a) Use gradient method to compute the directions $d^{(0)}, d^{(1)}, d^{(2)}, d^{(3)}$ and step-length: $\alpha_0, \alpha_1, \alpha_2, \alpha_3$
- b) Use Conjugate Gradient method to compute the directions $d^{(0)}, d^{(1)}, d^{(2)}, d^{(3)}$ and step-length: $\alpha_0, \alpha_1, \alpha_2, \alpha_3$
- c) In part (a) and (b), Compute the error bound $\|e^k\|_A$ in each step and compare with the theorem bound. [Hint: Create a table with 4 columns: exact error, computed error from theorem, computed error from part a, computed error from part b.]
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2) Show that β_k in conjugate gradient method can alternatively be computed as follows

$$\beta_k = \frac{\langle r^{k+1}, r^{k+1} \rangle}{\langle r^k, r^k \rangle}$$

3) In the Conjugate Gradient algorithm, show that value:

$$\beta_k = \frac{\langle r^{k+1}, r^{k+1} \rangle}{\langle r^k, r^k \rangle} \text{ is minimizing.}$$

[Hint: expand $\|\xi - \xi^k\|_A^2$ and use simple calculus]

4)) In the Conjugate Gradient algorithm, show that:

$$\langle d^k, r^k \rangle = \langle r^k, r^k \rangle$$
