King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 572 , Term: 081
Instructor: Dr. Faisal Fairag
Assignment (6)
Due Sunday 21/12/2008

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1) (Problem 5.1/Page 73) Prove (5.7).

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2) (Problem 5.6/Page 74)
Let $a(\cdot,\cdot)$ and $L(\cdot)$ satisfy the assumptions of the Lax-Milgram lemma. i.e.,

$$a(v,w) \le C_1 \|v\|_V \|w\|_V \qquad \forall v, w \in V,$$
$$a(v,v) \ge C_2 \|v\|_V^2 \qquad \forall v \in V,$$
$$L(v) \le C_3 \|v\|_V \qquad \forall v \in V,$$

Let $u \in V$ be the solution of

$$a(u,v) = L(v), \qquad \forall v \in V$$

Let $\hat{V} \subset V$ be a finite-dimensional subspace and let $\hat{u} \in \hat{V}$ be determined by Galerkin's method:

$$a(\hat{u},v) = L(v), \qquad \forall v \in \hat{V}$$

Prove that (note that $a(\cdot,\cdot)$ may be non-symmetric)

$$\|\hat{u} - u\|_V \le \frac{C_1}{C_2} \min_{\chi \in \hat{V}} \|\chi - u\|_V$$

Prove that, if is symmetric and , then

$$\|\hat{u} - u\|_a = \min_{\chi \in \hat{V}} \|\chi - u\|_a \qquad \text{and} \qquad \|\hat{u} - u\|_V \le \sqrt{\frac{C_1}{C_2}} \min_{\chi \in \hat{V}} \|\chi - u\|_V$$

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3) (Problem 5.7a/Page 74)
Consider the problem: $-\nabla \cdot (a\nabla u) + b \cdot \nabla u + cu = f$ in Ω, with $u = 0$ on Γ ,
Note that the presence of the convection term $b \cdot \nabla u$ makes the bilinear form non-symmetric.
(a) Formulate a finite element method for this problem and prove an error bound in the H_1-norm.
Hint: see Problem 5.6

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4) (Problem 5.9/Page 75)

Formulate a finite element problem corresponding to the nonhomogeneous Neuman problem (3.34). Prove error estimates.
