King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 572 , Term: 081
Instructor: Dr. Faisal Fairag
Assignment (4)
Due Sunday 9/11/2008

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(1) Prove that $T_{i,j}(u) = o(h^4)$ when r = 1/6 as indicated in the class note.

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(2) Consider

$$U_{i,j+1} = U_{i,j-1} + 2r(U_{i-1,j} - 2U_{i,j} + U_{i+1,j}) \qquad (3a)$$

(a) Show that (3a) is consistent with $u_t = u_{xx}$ with local truncation error of order $o(k^2 + h^2)$

(b) Show that (3a) is always unstable (for all r) [Hint: show eig ≥ 1]

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(3) Consider

$$U_{i,j+1} = U_{i,j-1} + 2r(U_{i-1,j} - 2U_{i,j} + U_{i+1,j}) \qquad (4a)$$

(a) Show that (4a) is consistent with $u_t = u_{xx}$ with local truncation error of order $o(k^2 + h^2 + (\frac{k}{h})^2)$

(b) Show that (4a) is always stable (for all r) [Hint: show eig ≤ 1]

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(4) Consider the following Problem:

$$\left.\begin{array}{ll} u_t = u_{xx} & t \ge 0 \\ u(0,t) = 0 & t \ge 0 \\ u(1,t) = 0 & t \ge 0 \\ u(x,0) = \begin{cases} 2x & 0 \le x \le \frac{1}{2} \\ 2(1-x) & \frac{1}{2} \le x \le 1 \end{cases} & \end{array}\right\} \quad (1a)$$

It is well known that (1a) is well posed with unique solution :

$$u(x,t) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}Sin(\frac{n\pi}{2})Sin(n\pi x)e^{-n^2\pi t}$$

(a) write a Matlab program to solve (1a) using the explicit method (2.1.5) with $h = 1/10$, $k = 1/1000$ and $T = 0.02$ and compare your results with the exact solution at $x = 0.5$. and store all j-levels in a matrix U(i,j).

(b) use *plot* command in Matlab to plot the graph at the level j=20.

(c) use movie command in Matlab to play the movie twenty times for j=0:20. [download the file record.m to see how to create a movie].
