

Example 2
part 11

Consider the constant coeff equation

$$au_x + by + cu = 0$$

a, b, c are numbers $a \neq 0$.

Find the general solutions of the PDE

$$\frac{dy}{dx} = \frac{b}{a}$$

(characteristic equation)

with general solution:

$$bx - ay = \xi \quad \text{--- (**)}$$

$$\eta(x, y) = \xi$$

use: $\xi = x$, $\eta = bx - ay$

with this transformation.

$$aw_\xi + cw = 0$$

$$\text{or } \boxed{w_\xi + \frac{c}{a}w = 0}$$

$$e^{\frac{c}{a}\xi} w = g(\eta)$$

$$\boxed{w(\xi, \eta) = e^{-\frac{c}{a}\xi} g(\eta)}$$

$$\boxed{u(x, y) = e^{-\frac{c}{a}x} g(bx - ay)}$$