Consider 1st order linear PDE (1.3)

$$a u_x + b u_y + c u = f \longrightarrow w_\xi + h w = F$$

assume $\xi = \varphi(x,y)$, $\eta = \psi(x,y)$

where $w(\xi,\eta) = u(x,y)$

we have: $u_x = w_\xi \xi_x + w_\eta \eta_x$

$u_y = w_\xi \xi_y + w_\eta \eta_y$ (1.5)

subsitute into (1.3):

$$(a\xi_x + b\xi_y) w_\xi + (a\eta_x + b\eta_y) w_\eta + cw = f$$

choose: $\eta = \psi(x,y)$ so that $a\eta_x + b\eta_y = 0$

- $\frac{\eta_x}{\eta_y} = -\frac{b}{a}$
- Suppose that $\eta(x,y) = c$ ——— (*)

$$d\eta = \eta_x dx + \eta_y dy = 0$$

implies that $\frac{dy}{dx} = -\frac{\eta_x}{\eta_y} = \frac{b}{a}$ ——— (1.6)

- it means that (*) is an integral of (1.6)
- (1.6) is called <u>characteristic equation</u> of (1.3)
- $\eta(x,y)$ defines a family of curves called <u>characteristic curves</u> or <u>characteristics</u>

- use: $\xi = x$, $\eta = \psi(x,y)$