King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 470 Exam II
Semester I, 2009- (091)
Dr. Faisal Fairag

ID:	
Name:	

Q		Points
1		20
2		15
3		20
4		13
5		20
6		12
Total		

Say Bismillah & Good luck

(1) State and prove the Maximum Principle Theorem for Harmonic Functions.

(2) Use separation of variable to solve the following equation

$$u_{tt} + 2u_t + u = c^2 u_{xx} \quad \text{for} \quad 0 < x < \pi, t > 0$$

in which c is positive constant. The boundary conditions are

$$u(0, t) = u(\pi, t) = 0 \quad \text{for} \quad t > 0$$

and the initial conditions are

$$u(x,0) = 1 \quad \text{and} \quad u_t(x,0) = -1 \quad \text{for} \quad 0 < x < \pi$$

(3) Consider the following problems

Problem 1:

$$u_{tt} = c^2 u_{xx} \quad \text{for} \quad -\infty < x < \infty, t > 0$$
$$u(x,0) = \sin x \ , \quad u_t(x,0) = \psi(x)$$

Problem 2:

$$u_{tt} = c^2 u_{xx} \quad \text{for} \quad -\infty < x < \infty, t > 0$$
$$u(x,0) = \sin x + 0.01 \ , \quad u_t(x,0) = \psi(x)$$

Let $u(t,x)$ **be a solution of problem 1 and**
$w(t,x)$ **be a solution of problem 2, then find an upper bound for the difference**

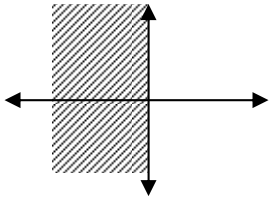
$$|v(t,x) - w(t,x)| \leq \text{.....}$$

(show all your work).

(4) Find the solution of the Drichlet problem for the left half-plane.

$$\Delta u = 0 \qquad \text{for} \quad -\infty < y < \infty, x < 0$$

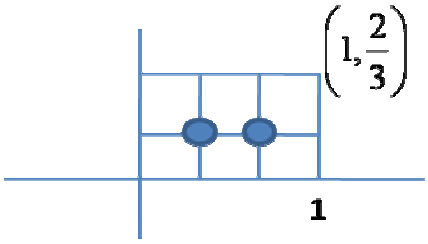
$$u(0, y) = f(y) \qquad \text{for} \quad -\infty < y < \infty$$



where

$$f(y) = \begin{cases} 0 & \text{for} \quad y < 0 \quad \text{and} \quad y > 1 \\ y & \text{for} \quad 0 \le y \le 1 \end{cases}$$

(5) use finite difference method to approximate the values $u\left(\frac{1}{3},\frac{1}{3}\right)$ and $u\left(\frac{2}{3},\frac{1}{3}\right)$ **of the solution of the Laplace equation**



$$\Delta u = f(x,y) \quad \text{for} \quad (x,y) \in \Omega$$
$$u = 0 \quad \text{for} \quad (x,y) \in \partial\Omega$$

where

$$\Omega = \left\{(x,y) : 0 < x < 1,\ 0 < y < \frac{2}{3}\right\}$$

$$f(x,y) = -2(x^2 + y^2)$$

(a) use the meshsize $h = \frac{1}{3}$**. Then write the linear system AU=b.**

(b) solve the linear system and write the approximate value of

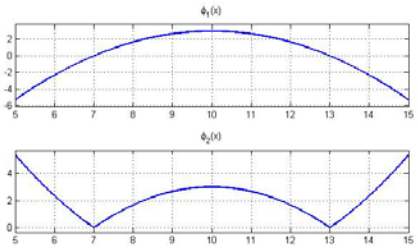
$u\left(\frac{1}{3},\frac{1}{3}\right) \cong$ .....

$u\left(\frac{2}{3},\frac{1}{3}\right) \cong$ .....

(6) Let w be a solution of thre problem

$$u_{tt} = u_{xx} \quad \text{for} \quad -\infty < x < \infty, t > 0$$

$$u(x, y) = \varphi_1(x), \quad u_t(x,0) = 0 \quad \text{for} \quad -\infty < x < \infty$$



Let u be a solution of

$$u_{tt} = u_{xx} \quad \text{for} \quad -\infty < x < \infty, t > 0$$

$$u(x, y) = \varphi_2(x), \quad u_t(x,0) = 0 \quad \text{for} \quad -\infty < x < \infty$$

where

$$\varphi_1(x) = -\frac{1}{3}x^2 + \frac{20}{3}x - \frac{91}{3}$$

$$\varphi_2(x) = \left| -\frac{1}{3}x^2 + \frac{20}{3}x - \frac{91}{3} \right|$$

Find: $\boxed{|v(10,3) - w(10,3)| = \text{.....}}$

(show all your work)