KEY KEY

MATH 301 **Term 061** **QUIZ 5**

1) Solve problem (1), section 12.4, page 672.

$$f(x) = \begin{cases} -1 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases} \qquad p = 2$$

$$c_n = \frac{1}{2p}\int_{-p}^{p} f(x)\, e^{\frac{-in\pi x}{p}}\, dx \quad 2 \checkmark$$

$$= \frac{1}{4}\int_{-2}^{2} f(x)\, e^{\frac{-in\pi x}{2}}\, dx = \frac{1}{4}\left[\int_{-2}^{0} + \int_{0}^{2}\right] \text{ — } (1)$$

$$\int_{-2}^{0} = \int_{-2}^{0} f(x)\, e^{\frac{-in\pi x}{2}}\, dx = -\int_{-2}^{0} e^{\frac{-in\pi x}{2}}\, dx$$

$$= -\left[-\frac{2}{in\pi} e^{\frac{-in\pi x}{2}}\right]_{-2}^{0} = \frac{2}{in\pi}\left[1 - e^{in\pi}\right] \quad 2 \text{ — } (2) \checkmark$$

$$\int_{0}^{2} = \int_{0}^{2} f(x)\, e^{\frac{-in\pi x}{2}}\, dx = \int_{0}^{2} e^{\frac{-in\pi x}{2}}\, dx$$

$$= \left[\frac{-2}{in\pi} e^{\frac{-in\pi x}{2}}\right]_{0}^{2} = \frac{-2}{in\pi}\left[e^{-in\pi} - 1\right] \quad 2 \text{ — } (3) \checkmark$$

$$(1),(2),(3) \Rightarrow c_n = \frac{1}{4}\left[\frac{2}{in\pi}\right]\left[1 - e^{in\pi} - e^{-in\pi} + 1\right]$$

$$= \frac{1}{4}\left(\frac{2}{in\pi}\right)\left[2 - (\cos n\pi + i\sin n\pi) - (\cos n\pi - i\sin n\pi)\right]$$

$$= \frac{1}{2in\pi}\left[2 - 2\cos n\pi\right] = \frac{1}{in\pi}(1 - \cos n\pi) \quad 4$$

$$= \frac{1}{in\pi}\left[1 - (-1)^n\right] = \frac{i}{-n\pi}\left[1 - (-1)^n\right] = \frac{i}{n\pi}\left((-1)^n - 1\right)$$

$n = \pm 1, \pm 2, \ldots$

$$2 \quad c_0 = \frac{1}{4}\int_{-2}^{2} f(x)\, e^{0}\, dx = \frac{1}{4}\int_{-2}^{2} f(x)\, dx = \frac{1}{4}\left[-\int_{-2}^{0} dx + \int_{0}^{2} dx\right]$$

$$= \frac{1}{4}\left[-(0+2) + (2-0)\right] = \frac{1}{4}(0) = 0 \quad \checkmark$$

$$\checkmark \quad f(x) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{i}{n\pi}\left[(-1)^n - 1\right] e^{in\pi x/2} \quad 3$$

2) The Fourier-Legendre series of f(x) on the interval (-1,1) is

$$f(x)=\sum_{n=0}^{\infty} c_n p_n(x).$$

Find c_6 for the function f(x)=1.

[Hint: $p_0(x), p_1(x), \ldots, p_5(x)$ is given in page 263].

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we have $(k+1)\, P_{k+1}(x) - (2k+1)\, x\, P_k(x) + k\, P_{k-1}(x) = 0$

let $k=5$ $\quad 6P_6 - 11xP_5 + 5P_4 = 0 \Rightarrow P_6 = \frac{1}{6}(11xP_5 - 5P_4)$

$$P_6(x) = \frac{1}{6}\left[\frac{11}{8}(63x^6 - 70x^4 + 15x^2) - \frac{5}{8}(35x^4 - 30x^2 + 3)\right]$$

$$= \frac{1}{48}\left[(693x^6 - 770x^4 + 165x^2) - (175x^4 - 150x^2 + 15)\right]$$

$$= \frac{1}{48}\left[693x^6 - 945x^4 + 315x^2 - 15\right]$$

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we have $C_n = \frac{2n+1}{2}\int_{-1}^{1} f(x)\, P_n(x)\, dx$

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$$C_6 = \frac{13}{2}\int_{-1}^{1} f(x)\, P_6(x)\, dx = \frac{13}{2}\int_{-1}^{1} P_6(x)\, dx$$

$$= \frac{13}{2}\cdot\frac{1}{48}\int_{-1}^{1}\underbrace{(693x^6 - 945x^4 + 315x^2 - 15)}_{\text{even function}}\, dx$$

$$= \left(\frac{13}{2}\cdot\frac{1}{48}\right) 2\int_{0}^{1}(693x^6 - 945x^4 + 315x^2 - 15)\, dx$$

$$= \frac{13}{48}\left[\frac{693x^7}{7} - \frac{945x^5}{5} + \frac{315x^3}{3} - 15x\right]_0^1$$

$$= \frac{13}{48}\left[\left(\frac{\cancel{693}^{99}}{\cancel{7}} - \frac{\cancel{945}^{189}}{\cancel{5}_1} + \frac{\cancel{315}^{105}}{\cancel{3}_1} - 15\right) - 0\right]$$

$$= \frac{13}{48}\left[99 - 189 + 105 - 15\right] = \frac{13}{48}[0] = 0$$

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