Name:	ID:	serial:

MATH 301 **Term 061** **QUIZ 3**

1) Use the Laplace transform to solve the initial value problem:

$$y'' - y' = e^t \cos t$$
$$y(0) = 0$$
$$y'(0) = 0$$

$$\mathcal{L}[y''] - \mathcal{L}[y'] = \mathcal{L}[e^t \cos t]$$

$$\{s^2 Y(s) - s y(0) - y'(0)\} - \{s Y(s) - y(0)\} = \frac{s}{s^2+1}\Big|_{s \to s-1}$$ 3

$$s^2 Y(s) - s Y(s) = \frac{(s-1)}{(s-1)^2+1}$$

$$Y(s) = \frac{(s-1)}{[(s-1)^2+1][s^2-s]} = \frac{\cancel{(s-1)}}{s[(s-1)^2+1]\cancel{[s-1]}}$$

$$Y(s) = \frac{1}{s[(s-1)^2+1]}$$ 3

By partial fraction:

$$Y(s) = \frac{1}{s[(s-1)^2+1]} = \frac{1}{s(s^2-2s+2)} = \frac{1/2}{s} + \frac{-\frac{1}{2}s+1}{(s-1)^2+1}$$

$$= \frac{1/2}{s} - \frac{1}{2}\frac{s-1}{(s-1)^2+1} + \frac{1}{2}\frac{1}{(s-1)^2+1}$$ 4

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \frac{1}{2} - \frac{1}{2}e^t \cos t + \frac{1}{2}e^t \sin t$$ 5

2) Use the Laplace transform to solve the initial value problem:

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$y'' + y' = f(t)$

$y(0) = 0$

$y'(0) = 1$

$f(t) = [\mathcal{U}(t-\pi) - \mathcal{U}(t-2\pi)]$ 2

when $f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$

$\mathcal{L}[y''] + \mathcal{L}[y'] = \mathcal{L}[f(t)]$

$s^2 Y(s) - s\,y(0) - y'(0) + sY(s) - y(0) = \mathcal{L}[f(t)]$

$s^2 Y(s) - 1 + sY(s) = \mathcal{L}[\mathcal{U}(t-\pi)] - \mathcal{L}[\mathcal{U}(t-2\pi)]$

$Y(s)[s^2 + s] - 1 = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}$

$Y(s) = \frac{e^{-\pi s} - e^{-2\pi s}}{s^2(s+1)} + \frac{1}{s(s+1)}$ 3

by partial fraction:

$\frac{1}{s(s+1)} = \frac{1}{s} + \frac{-1}{s+1}$ and $\frac{1}{s^2(s+1)} = \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}$

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$\mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right] = 1 - e^{-t}$ — 1 $\mathcal{L}\left[\frac{1}{s^2(s+1)}\right] = -1 + t + e^{-t}$

$\mathcal{L}^{-1}\left[e^{-\pi s}\frac{1}{s^2(s+1)}\right] = \left[-1 + (t-\pi) + e^{-(t-\pi)}\right]\mathcal{U}(t-\pi)$ — 2 3

$\mathcal{L}^{-1}\left[e^{-2\pi s}\frac{1}{s^2(s+1)}\right] = \left[-1 + (t-2\pi) + e^{-(t-2\pi)}\right]\mathcal{U}(t-2\pi)$ — 3 3

$\Rightarrow y(t) =$ RHS 1 + RHS 2 + RHS 3 1