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MATH 301

Term 061

QUIZ 1

1) Given that  $f(x,y,z) = \frac{xy^2}{z^3}$  compute the gradient.

(HW: #3/page475)

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}\right)i + \left(\frac{\partial f}{\partial y}\right)j + \left(\frac{\partial f}{\partial z}\right)k$$

$$= \left(\frac{y^2}{z^3}\right)i + \left(\frac{2xy}{z^3}\right)j + \left(\frac{-3xy^2}{z^4}\right)k$$

$$\frac{\partial f}{\partial x} = \frac{y^2}{z^3}$$

$$\frac{\partial f}{\partial y} = \frac{2xy}{z^3}$$

$$\frac{\partial f}{\partial z} = -3 \frac{xy^2}{z^4}$$

2) Find the directional derivative of the function  $f(x,y) = \frac{xy}{x+y}$  at the point  $(2,-1)$  in the direction of the vector  $v = 6i + 8j$ .

(#13/page475)

$$\frac{\partial f}{\partial x} = \frac{y(x+y) - xy}{(x+y)^2} = \frac{y^2}{(x+y)^2}, \quad \frac{\partial f}{\partial y} = \frac{x(x+y) - xy}{(x+y)^2} = \frac{x^2}{(x+y)^2}$$

$$\frac{\partial f}{\partial x}(2,-1) = \frac{1}{1} = 1, \quad \frac{\partial f}{\partial y}(2,-1) = \frac{4}{1} = 4$$

$$\nabla f(2,-1) = i + 4j \quad \text{--- (1)}$$

$$\|v\| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

$u =$  a unit vector in the direction of  $v = \frac{6}{10}i + \frac{8}{10}j$  --- (2)

from (1) and (2)

$$\begin{aligned} \text{The directional derivative} &= \nabla f(2,-1) \cdot u = (i+4j) \cdot \left(\frac{6}{10}i + \frac{8}{10}j\right) \\ &= \frac{6}{10} + \frac{32}{10} = \frac{38}{10} = 3.8 \end{aligned}$$

$$D_u f(2,-1) = 3.8$$