Name: KEY	**ID:** KEY	**serial:**

MATH 301 **Term 061** **QUIZ 4**

1) Verify Stokes theorem: Assume that the surface S is oriented upward.
$F = zi + xj + yk$; S that portion of the plane $2x + y + 2z = 6$ in the first octant.

$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ z & x & y \end{vmatrix} = i + j + k$ 2. Let $g = 2x + y + 2z - 6 \Rightarrow$
$\nabla g = 2i + j + 2k \Rightarrow \|\nabla g\| = 3$

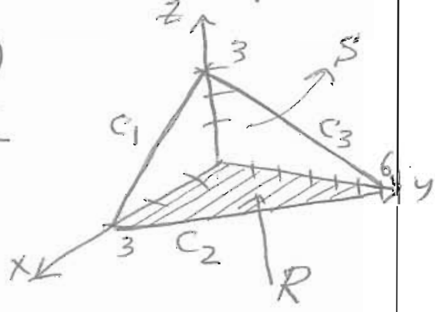
$n = \frac{\nabla g}{\|\nabla g\|} = \frac{2}{3}i + \frac{1}{3}j + \frac{2}{3}k \Rightarrow \iint_S (\text{curl } F)\cdot n\, ds = \iint_S \left(\frac{2}{3} + \frac{1}{3} + \frac{2}{3}\right) ds =$
$= \iint_S \frac{5}{3}\, ds$. 2

The surface S is given by $z = 3 - \frac{1}{2}y - x \Rightarrow \frac{\partial z}{\partial x} = -1, \ \frac{\partial z}{\partial y} = -\frac{1}{2}$

Hence $dS = \sqrt{1 + (-1)^2 + (-\frac{1}{2})^2}\, dA = \frac{3}{2}\, dA$. 2

Now, $\iint_S (\text{curl } F)\cdot n\, ds = \iint_S \frac{5}{3}\, ds = \iint_R \left(\frac{5}{3}\right)\left(\frac{3}{2}\right) dA = \frac{5}{2}\iint_R dA$

$= \frac{5}{2}(\text{Area of } R) = \frac{5}{2}(\text{the triangle})$
$= \frac{5}{2}\left(\frac{1}{2} \times 6 \times 3\right) = \frac{5}{2}(9) = \frac{45}{2}$ 2



line integral

C_1	C_2	C_3
$x = t \quad 0 \le t \le 3$	$x = t \quad 0 \le x \le 3$	$x = 0$
$y = 0$	$y = 6 - 2t$	$y = t \quad 0 \le y \le 6$
$z = 3 - t$	$z = 0$	$z = 3 - \frac{1}{2}t$

$\oint_{C_1} F\cdot dr = \int_{C_1} (3-t)\, dt = \int_0^3 (3-t)\, dt = \frac{9}{2}$ 2

$\oint_{C_2} F\, dr = \int_{C_2} t(-2dt) = \int_3^0 -2t\, dt = 9$ 2

$\oint_{C_3} F\, dr = \int_{C_3} t\left(-\frac{1}{2}\right) dt = \int_6^0 -\frac{1}{2}t\, dt = -\frac{1}{4}(0 - 36) = 9$ 2

$\oint_C z\,dx + x\,dy + y\,dz = \oint_{C_1} + \oint_{C_2} + \oint_{C_3} = \frac{9}{2} + 9 + 9 = \frac{45}{2}$ 1

2) Use the <u>Divergence Theorem</u> to find the outward flux $\iint_S (F \cdot n) ds$ of the vector field

$F = 4xi + yj + 4zk$,

D the region bounded by the sphere $x^2 + y^2 + z^2 = 4$

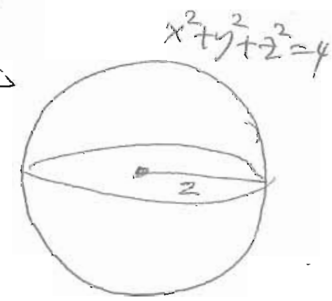
$$\text{div } F = \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(4z) = 4 + 1 + 4 = 9 \quad 2$$

by Divergence Theorem

$$\iint_S (F \cdot n)\, ds = \iiint_D \text{div} F\, dV = \iiint_D 9\, dV \quad 2$$

$$= 9 \iiint_D dV = 9 \times (\text{volume of the sphere})$$

$$= 9 \times \left(\frac{4}{3}\pi 2^3\right) = 96\pi \quad 3$$



3) **(True or False)**	
I) The functions $f_1(x) = x^2$ and $f_2(x) = x^3$ are orthogonal on [-2,2]	(T) 2
II) $\left\{ \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}x \right\}$ is an <u>orthonormal set</u> on the interval [-1,1]	(T) 2
III) $\lVert f(x) \rVert^2 = \frac{3}{28}$ where $f(x) = x + 1$	(F) 2
IV) The orthogonal set {1, sinx, sin2x, sin3x, ….} is <u>complete</u>.	(F) 2