Name: KEY	**ID:** KEY	**serial:**

MATH 301 **Term 061** **QUIZ 2**

1) Find the Curl and the divergence $F(x,y,z)=(xz)i+(yz)j+(xy)k$

$$div\ F(x,y,z)=\left(\frac{\partial P}{\partial x}\right)+\left(\frac{\partial Q}{\partial y}\right)+\left(\frac{\partial R}{\partial z}\right)=z+z+0=2z$$ 3

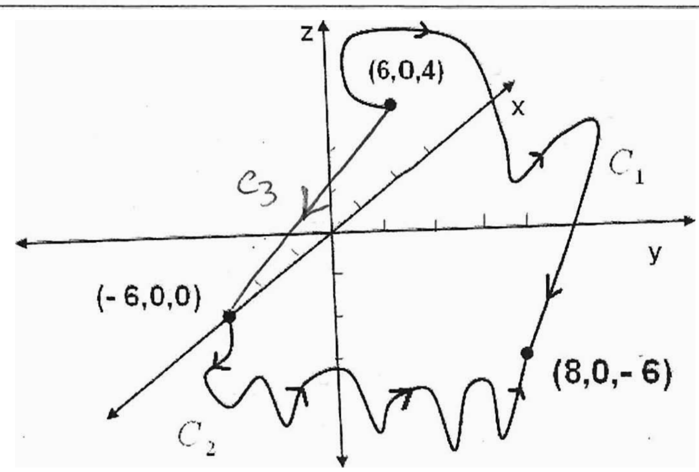
$$\text{curl}\ F(x,y,z)=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & yz & xy \end{vmatrix}=(x-y)\,i+(x-y)\,j+(0-0)k$$

$$=(x-y)\,i+(x-y)\,j$$ 3

2)Given that:

$$\int (y+z)dx+(x+z)dy+(y+x)dz=-72$$

$$\int (y+z)dx+(x+z)dy+(y+x)dz=-48$$



Where C_1 is the curve starts at the point (6,0,4) and ends at the point (8,0,-6) and C_2 is the curve starts at the point (-6,0,0) and ends at the point (8,0,-6) as shown in the figure.

Evaluate the line integral

$$\int_{(6,0,4)}^{(-6,0,0)} (y+z)dx+(x+z)dy+(y+x)dz=$$

Let C_3 be the line connecting the points (6,0,4) and (-6,0,0)

C be the closed curve $C=C_3\cup(C_2)\cup(-C_1)$

$P(x,y,z)=y+z$; $Q(x,y,z)=x+z$; $R(x,y,z)=y+x$

$\frac{\partial P}{\partial y}=1$, $\frac{\partial P}{\partial z}=1$; $\frac{\partial Q}{\partial x}=1$, $\frac{\partial Q}{\partial z}=1$; $\frac{\partial R}{\partial y}=1$, $\frac{\partial R}{\partial x}=1$ 3

Note that $\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y}$, $\frac{\partial R}{\partial y}=\frac{\partial Q}{\partial z}$, $\frac{\partial P}{\partial z}=\frac{\partial R}{\partial x}$ 3 $\Rightarrow \oint_C$ is indep. of the path

Hence $0=\oint_C=\int_{C_3}+\int_{C_2}-\int_{C_1}\Rightarrow\int_{C_3}=\int_{C_1}-\int_{C_2}=-72+48=-24$

$$\Rightarrow\int_{(6,0,4)}^{(-6,0,0)}=\int_{C_3}=-24$$ 6

3) Show that the given integral is independent of the path. Then Evaluate (#24/pp 502

$$\int_{(-2,3,1)}^{(0,0,0)} \underbrace{(2xz)}_{P(x,y,z)}dx + \underbrace{(2yz)}_{Q(x,y,z)}dy + \underbrace{(x^2+y^2)}_{R(x,y,z)}dz =$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 0 \quad , \frac{\partial R}{\partial y} = \frac{\partial Q}{\partial z} = 2y \quad , \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} = 2x$$

3

$\Rightarrow$ $P\,dx + Q\,dy + R\,dz$ is an exact differential and the line integral is indep. of the path.

Finding $\varphi(x,y,z)$:

step 1: $\frac{\partial \varphi}{\partial x} = P(x,y,z) = 2xz$

$$\varphi(x,y,z) = \int 2xz\,dx + g(y,z) = x^2z + g(y,z) \quad \text{——— (1)}$$

step 2: $\frac{\partial \varphi}{\partial y} = 0 + \frac{\partial g}{\partial y}(y,z) = Q(x,y,z) = 2yz$

$$\Rightarrow \frac{\partial g}{\partial y}(y,z) = 2yz$$

$$\Rightarrow g(y,z) = \int 2yz\,dy + f(z) = y^2z + f(z) \quad \text{——— (2)}$$

$$(1) \text{ \& } (2) \Rightarrow \varphi(x,y,z) = x^2z + y^2z + f(z) \quad \text{——— (3)}$$

Step 3: $\frac{\partial \varphi}{\partial z} = x^2 + y^2 + f'(z) = R(x,y,z) = x^2 + y^2$

$$\Rightarrow f'(z) = 0 \Rightarrow f(z) = \text{constant}. \text{ (choose 0)}$$

Hence $\boxed{\varphi(x,y,z) = x^2z + y^2z}$ 6

Finding $\int_{(-2,3,1)}^{(0,0,0)}$: $\varphi(0,0,0) = 0$, $\varphi(-2,3,1) = 4(1) + 9(1) = 13$

$$\int_{(-2,3,1)}^{(0,0,0)} = \varphi(0,0,0) - \varphi(-2,3,1) = 0 - 13 = -13$$

$$\boxed{\int_{(-2,3,1)}^{(0,0,0)} = -13}$$ 3