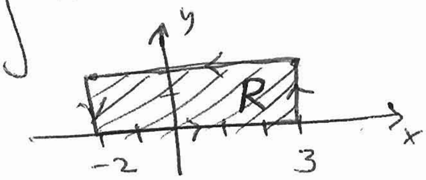
§(9.12) : Green's Theorem.

#8/p520) $P(x,y) = x - 3y$, $Q(x,y) = 4x + y$ } all cont.

$\dfrac{\partial P}{\partial y} = -3$, $\dfrac{\partial Q}{\partial x} = 4$



By Green's theorem,

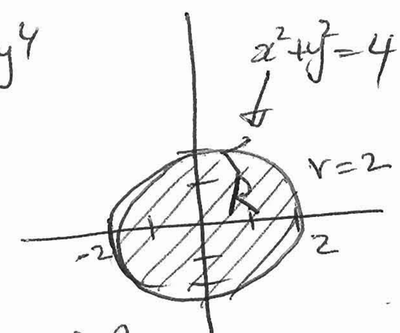
$$\oint_C (x-3y)\,dx + (4x+y)\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_R (4+3)\,dA = 7\iint_R dA = 7(\text{Area of } R)$$

$$= 7(\text{Area of the rectangle}) = 7(5\times 2) = 70$$

#7/p520) $P(x,y) = x^4 - 2y^3$, $Q(x,y) = 2x^3 - y^4$

$\dfrac{\partial P}{\partial y} = -6y^2$, $\dfrac{\partial Q}{\partial x} = 6x^2$



Using Green's theorem,

$$\oint_C (x^4 - 2y^3)\,dx + (2x^3 - y^4)\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_R (6x^2 + 6y^2)\,dA$$

$$= \int_0^{2\pi}\int_0^2 6r^2 \cdot r \cdot dr \cdot d\theta \quad \text{(using polar Coordinates)}$$

$$= \int_0^{2\pi}\int_0^2 6r^3\,dr\,d\theta = \int_0^{2\pi} \left[\frac{3}{2}r^4\right]_0^2 d\theta$$

$$= \int^{2\pi} [24 - 0]\,d\theta = 24\int_0^{2\pi} d\theta = 24(2\pi) = 48\pi$$