

(7) $\oint_C x dy = - \oint_C y dx = \text{area of } R$

For the first integral

$$\oint_C x dy = \oint_C 0 \cdot dx + x \cdot dy \quad \begin{array}{l} P=0, P_y=0 \\ Q=x, Q_x=1 \end{array}$$

Now apply Green's on the first integral

$$\oint_C x dy = \iint_R (Q_x - P_y) dA = \iint_R 1 \cdot dA$$

= area of R .

$$\overline{- \oint_C y dx = \oint_C -y dx + 0 \cdot dy}$$

$$P = -y \rightarrow P_y = -1$$

$$Q = 0 \rightarrow Q_x = 0$$

$$- \oint_C y dx = \iint_R [0 - (-1)] dA = \iint_R dA$$