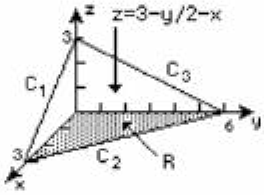
# Sec 9.14

**3. Surface Integral:** curl $\mathbf{F} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Letting $g(x, y, z) = 2x + y + 2z - 6$, we have $\nabla g = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{n} = (2\mathbf{i} + \mathbf{j} + 2\mathbf{k})/3$. Then $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS = \iint_S \frac{5}{3}\, dS$. Letting the surface be $z = 3 - \frac{1}{2}y - x$ we have $z_x = -1$, $z_y = -\frac{1}{2}$, and $dS = \sqrt{1 + (-1)^2 + (-\frac{1}{2})^2}\, dA = \frac{3}{2}\, dA$. Then




$$\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS = \iint_R \frac{5}{3}\left(\frac{3}{2}\right) dA = \frac{5}{2} \times (\text{area of } R) = \frac{5}{2}(9) = \frac{45}{2}.$$

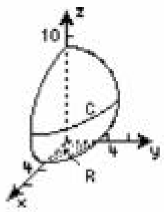
**Line Integral:** $C_1$: $z = 3 - x$, $0 \le x \le 3$, $y = 0$; $C_2$: $y = 6 - 2x$, $3 \ge x \ge 0$, $z = 0$; $C_3$: $z = 3 - y/2$, $6 \ge y \ge 0$, $x = 0$.

$$\begin{aligned}
\oint_C z\,dx + x\,dy + y\,dz &= \iint_{C_1} z\,dx + \int_{C_2} x\,dy + \int_{C_3} y\,dz \\
&= \int_0^3 (3 - x)\,dx + \int_3^0 x(-2\,dx) + \int_6^0 y(-dy/2) \\
&= \left(3x - \frac{1}{2}x^2\right)\Big|_0^3 - x^2\Big|_3^0 - \frac{1}{4}y^2\Big|_6^0 = \frac{9}{2} - (0 - 9) - \frac{1}{4}(0 - 36) = \frac{45}{2}
\end{aligned}$$

**6.** curl $\mathbf{F} = -2xz\mathbf{i} + z^2\mathbf{k}$. A unit vector normal to the plane is $\mathbf{n} = (\mathbf{j} + \mathbf{k})/\sqrt{2}$. From $z = 1 - y$, we have $z_x = 0$ and $z_y = -1$. Thus, $dS = \sqrt{1 + 1}\, dA = \sqrt{2}\, dA$ and

$$\begin{aligned}
\oint_C \mathbf{F} \cdot d\mathbf{r} &= \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS = \iint_R \frac{1}{\sqrt{2}} z^2 \sqrt{2}\, dA = \iint_R (1 - y)^2\, dA \\
&= \int_0^2 \int_0^1 (1 - y)^2\, dy\, dx = \int_0^2 -\frac{1}{3}(1 - y)^3 \Big|_0^1 dx = \int_0^2 \frac{1}{3}\, dx = \frac{2}{3}.
\end{aligned}$$

**14.** Parameterize $C$ by $x = 5\cos t$, $y = 5\sin t$, $z = 4$, for $0 \le t \le 2\pi$. Then,




$$\begin{aligned}
\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS &= \oint_C \mathbf{F} \cdot \mathbf{r} = \oint_C y\,dx + (y - x)\,dy + z^2\,dz \\
&= \int_0^{2\pi} [(5\sin t)(-5\sin t) + (5\sin t - 5\cos t)(5\cos t)]\, dt \\
&= \int_0^{2\pi} (25\sin t\cos t - 25)\, dt = \left(\frac{25}{2}\sin^2 t - 25t\right)\Big|_0^{2\pi} = -50\pi.
\end{aligned}$$

**17.** We take the surface to be $z = 0$. Then $\mathbf{n} = \mathbf{k}$ and $dS = dA$. Since curl $\mathbf{F} = \dfrac{1}{1+y^2}\,\mathbf{i} + 2ze^{x^2}\mathbf{j} + y^2\mathbf{k}$,

$$\begin{aligned}\oint_C z^2 e^{x^2}\,dx + xy\,dy + \tan^{-1} y\,dz &= \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\,dS = \iint_S y^2\,dS = \iint_R y^2\,dA \\ &= \int_0^{2\pi}\int_0^3 r^2 \sin^2\theta\, r\,dr\,d\theta = \int_0^{2\pi} \frac{1}{4} r^4 \sin^2\theta \,\Big|_0^3\, d\theta \\ &= \frac{81}{4}\int_0^{2\pi} \sin^2\theta\,d\theta = \frac{81\pi}{4}\,.\end{aligned}$$