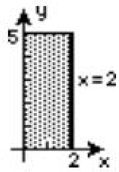
SEC 9.13

3. Using $f(x,y)=z=\sqrt{16-x^2}$ we see that for $0\le x\le 2$ and $0\le y\le 5$, $z>0$.

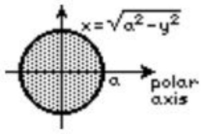


Thus, the surface is entirely above the region. Now $f_x=-\dfrac{x}{\sqrt{16-x^2}}$, $f_y=0$,

$1+f_x^2+f_y^2=1+\dfrac{x^2}{16-x^2}=\dfrac{16}{16-x^2}$ and

$$A=\int_0^5\int_0^2\frac{4}{\sqrt{16-x^2}}\,dx\,dy=4\int_0^5\sin^{-1}\frac{x}{4}\bigg|_0^2 dy=4\int_0^5\frac{\pi}{6}\,dy=\frac{10\pi}{3}.$$

11. There are portions of the surface in each octant with areas equal to the area of the portion in the first octant. Using $f(x,y)=z=\sqrt{a^2-y^2}$ we have $f_x=0$, $f_y=\dfrac{y}{\sqrt{a^2-y^2}}$,



$1+f_x^2+f_y^2=\dfrac{a^2}{a^2-y^2}$. Then

$$A=8\int_0^a\int_0^{\sqrt{a^2-y^2}}\frac{a}{\sqrt{a^2-y^2}}\,dx\,dy=8a\int_0^a\frac{x}{\sqrt{a^2-y^2}}\bigg|_0^{\sqrt{a^2-y^2}}dy=8a\int_0^a dy=8a^2.$$

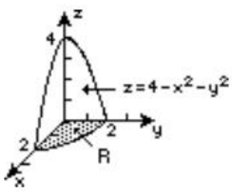
26. Write the equation of the surface as $x=6-2y-3z$. Then $x_y=-2$, $x_z=-3$; $dS=\sqrt{1+4+9}=\sqrt{14}$.

$$\iint_S(3z^2+4yz)\,dS=\int_0^2\int_0^{3-3z/2}(3z^2+4yz)\sqrt{14}\,dy\,dz=\sqrt{14}\int_0^2(3yz+2y^2z)\bigg|_0^{3-3z/2}dz$$

$$=\sqrt{14}\int_0^2\left[9z\left(1-\frac{z}{2}\right)+18z\left(1-\frac{z}{2}\right)^2\right]dz=\sqrt{14}\int_0^2\left(27z-\frac{45}{2}z^2+\frac{9}{2}z^3\right)dz$$

$$=\sqrt{14}\left(\frac{27}{2}z^2-\frac{15}{2}z^3+\frac{9}{8}z^4\right)\bigg|_0^2=\sqrt{14}(54-60+18)=2\sqrt{14}$$

33. The surface is $g(x,y,z)=x^2+y^2+z-4$. $\nabla g=2x\mathbf{i}+2y\mathbf{j}+\mathbf{k}$,



$|\nabla g|=\sqrt{4x^2+4y^2+1}$; $\mathbf{n}=\dfrac{2x\mathbf{i}+2y\mathbf{j}+\mathbf{k}}{\sqrt{4x^2+4y^2+1}}$; $\mathbf{F}\cdot\mathbf{n}=\dfrac{x^3+y^3+z}{\sqrt{4x^2+4y^2+1}}$;

$z_x=-2x$, $z_y=-2y$, $dS=\sqrt{1+4x^2+4y^2}\,dA$. Using polar coordinates,

$$\text{Flux}=\iint_S\mathbf{F}\cdot\mathbf{n}\,dS=\iint_R(x^3+y^3+z)\,dA=\iint_R(4-x^2-y^2+x^3+y^3)\,dA$$

$$=\int_0^{2\pi}\int_0^2(4-r^2+r^3\cos^3\theta+r^3\sin^3\theta)\,r\,dr\,d\theta$$

$$=\int_0^{2\pi}\left(2r^2-\frac{1}{4}r^4+\frac{1}{5}r^5\cos^3\theta+\frac{1}{5}r^5\sin^3\theta\right)\bigg|_0^2 d\theta$$

$$=\int_0^{2\pi}\left(4+\frac{32}{5}\cos^3\theta+\frac{32}{5}\sin^3\theta\right)d\theta=4\theta\bigg|_0^{2\pi}+0+0=8\pi.$$