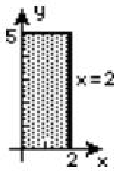
SEC 9.13

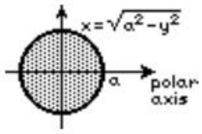
3. Using $f(x,y) = z = \sqrt{16 - x^2}$ we see that for $0 \le x \le 2$ and $0 \le y \le 5$, $z > 0$. Thus, the surface is entirely above the region. Now $f_x = -\dfrac{x}{\sqrt{16 - x^2}}$, $f_y = 0$,



$1 + f_x^2 + f_y^2 = 1 + \dfrac{x^2}{16 - x^2} = \dfrac{16}{16 - x^2}$ and

$$A = \int_0^5 \int_0^2 \frac{4}{\sqrt{16 - x^2}}\, dx\, dy = 4\int_0^5 \sin^{-1}\frac{x}{4}\Big|_0^2 dy = 4\int_0^5 \frac{\pi}{6}\, dy = \frac{10\pi}{3}.$$

11. There are portions of the surface in each octant with areas equal to the area of the portion in the first octant. Using $f(x,y) = z = \sqrt{a^2 - y^2}$ we have $f_x = 0$, $f_y = \dfrac{y}{\sqrt{a^2 - y^2}}$,



$1 + f_x^2 + f_y^2 = \dfrac{a^2}{a^2 - y^2}$. Then

$$A = 8\int_0^a \int_0^{\sqrt{a^2-y^2}} \frac{a}{\sqrt{a^2 - y^2}}\, dx\, dy = 8a\int_0^a \frac{x}{\sqrt{a^2 - y^2}}\Big|_0^{\sqrt{a^2-y^2}} dy = 8a\int_0^a dy = 8a^2.$$

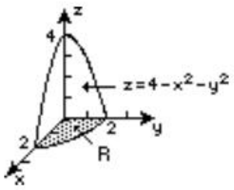
26. Write the equation of the surface as $x = 6 - 2y - 3z$. Then $x_y = -2$, $x_z = -3$; $dS = \sqrt{1 + 4 + 9} = \sqrt{14}$.

$$\iint_S (3z^2 + 4yz)\, dS = \int_0^2 \int_0^{3-3z/2} (3z^2 + 4yz)\sqrt{14}\, dy\, dz = \sqrt{14}\int_0^2 (3yz + 2y^2z)\Big|_0^{3-3z/2} dz$$

$$= \sqrt{14}\int_0^2 \left[9z\left(1 - \frac{z}{2}\right) + 18z\left(1 - \frac{z}{2}\right)^2\right] dz = \sqrt{14}\int_0^2 \left(27z - \frac{45}{2}z^2 + \frac{9}{2}z^3\right) dz$$

$$= \sqrt{14}\left(\frac{27}{2}z^2 - \frac{15}{2}z^3 + \frac{9}{8}z^4\right)\Big|_0^2 = \sqrt{14}(54 - 60 + 18) = 2\sqrt{14}$$

33. The surface is $g(x,y,z) = x^2 + y^2 + z - 4$. $\nabla g = 2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k}$,



$|\nabla g| = \sqrt{4x^2 + 4y^2 + 1}$; $\mathbf{n} = \dfrac{2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k}}{\sqrt{4x^2 + 4y^2 + 1}}$; $\mathbf{F}\cdot\mathbf{n} = \dfrac{x^3 + y^3 + z}{\sqrt{4x^2 + 4y^2 + 1}}$;

$z_x = -2x$, $z_y = -2y$, $dS = \sqrt{1 + 4x^2 + 4y^2}\, dA$. Using polar coordinates,

$$\text{Flux} = \iint_S \mathbf{F}\cdot\mathbf{n}\, dS = \iint_R (x^3 + y^3 + z)\, dA = \iint_R (4 - x^2 - y^2 + x^3 + y^3)\, dA$$

$$= \int_0^{2\pi} \int_0^2 (4 - r^2 + r^3\cos^3\theta + r^3\sin^3\theta)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(2r^2 - \frac{1}{4}r^4 + \frac{1}{5}r^5\cos^3\theta + \frac{1}{5}r^5\sin^3\theta\right)\Big|_0^2 d\theta$$

$$= \int_0^{2\pi} \left(4 + \frac{32}{5}\cos^3\theta + \frac{32}{5}\sin^3\theta\right) d\theta = 4\theta\Big|_0^{2\pi} + 0 + 0 = 8\pi.$$