SEC 4.5

4. The Laplace transform of the differential equation yields

$$\mathscr{L}\{y\} = \frac{1}{4}\,\frac{4}{s^2+16}e^{-2\pi s}$$

so that

$$y = \frac{1}{4}\sin 4(t-2\pi)\,\mathscr{U}(t-2\pi) = \frac{1}{4}\sin 4t\,\mathscr{U}(t-2\pi).$$

8. The Laplace transform of the differential equation yields

$$\mathscr{L}\{y\} = \frac{s+1}{s^2(s-2)} + \frac{1}{s(s-2)}\,e^{-2s} = \frac{3}{4}\,\frac{1}{s-2} - \frac{3}{4}\,\frac{1}{s} - \frac{1}{2}\,\frac{1}{s^2} + \left[\frac{1}{2}\,\frac{1}{s-2} - \frac{1}{2}\,\frac{1}{s}\right]e^{-2s}$$

so that

$$y = \frac{3}{4}e^{2t} - \frac{3}{4} - \frac{1}{2}t + \left[\frac{1}{2}e^{2(t-2)} - \frac{1}{2}\right]\mathscr{U}(t-2).$$

12. The Laplace transform of the differential equation yields

$$\begin{aligned}\mathscr{L}\{y\} &= \frac{1}{(s-1)^2(s-6)} + \frac{e^{-2s}+e^{-4s}}{(s-1)(s-6)} \\ &= -\frac{1}{25}\,\frac{1}{s-1} - \frac{1}{5}\,\frac{1}{(s-1)^2} + \frac{1}{25}\,\frac{1}{s-6} + \left[-\frac{1}{5}\,\frac{1}{s-1} + \frac{1}{5}\,\frac{1}{s-6}\right]\left(e^{-2s}+e^{-4s}\right)\end{aligned}$$

so that

$$y = -\frac{1}{25}e^{t} - \frac{1}{5}te^{t} + \frac{1}{25}e^{6t} + \left[-\frac{1}{5}e^{t-2} + \frac{1}{5}e^{6(t-2)}\right]\mathscr{U}(t-2) + \left[-\frac{1}{5}e^{t-4} + \frac{1}{5}e^{6(t-4)}\right]\mathscr{U}(t-4).$$